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THE COMBINED PART SELECTION AND MACHINE LOADING PROBLEMS
IN FLEXIBLE MANUFACTURING SYSTEMS

by
Ming Liang

A Dissertation
Submitted to the Faculty of Graduate Studies and Research
through the Department of Industrial Engineering in
Partial Fulfillment of the Requirements for the
University of Windsor

Windsor, Ontario, Canada

1991



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ABSTRACT

Part selection (PS) and machine loading (ML) are two major interrelated production planning problems in flexible manufacturing systems (FMSs). The two problems have been treated separately in most previous research work, thereby raising the possibility of conflicts and inconsistencies between the two sets of individually obtained solutions (Hwang & Shogan, 1989). This research is directed towards the development of an integrated approach to solve the combined part selection and machine loading problems in FMSs to obtain consistent solutions.

Three classes of FMSs are identified: FMS I, FMS II and the hybrid system (HS). FMSs I and II are identified based on tooling strategies. The third class of FMSs is a hybrid system where conventional manufacturing systems (CMS) and FMS sub-systems exist concurrently. Decisions related to part selection, job allocation, tool selection and assignment and process route selection are considered for both FMSs I and II. In the hybrid system, sharing of loads between CMS and FMS sub-systems is further investigated, in addition to the decision modules mentioned above.

Associated bicriteria models are developed for the combined PS and ML problems in the three classes of FMSs to take advantage of both productivity and flexibility that FMS can offer. The models, once solved, will yield a set of consistent solutions for both PS and ML problems. Example problems are solved for all of the developed models. The benefits of considering secondary objectives, and the

effects of magazine capacity and available machining time on PS decision are discussed.

The solution algorithms for the models associated with FMSs I and II have been proposed through the Lagrangian relaxation based method incorporating the decomposition principle and column generation scheme. The FORTRAN 77 program is coded for a representative model (Model M-1). Computational experience is presented for several sets of test problems on IBM 4381 mainframe. The efficiency of the algorithm and FORTRAN code are demonstrated through the test problems.

DEDICATION

To my parents

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NOTATIONS

Subscripts

$i = 1, \dots, I$ part type

$j = 1, \dots, J_p$ operation

$k, l = 1, \dots, K$ machining center (in FMS), or machine (or work center)
(in CMS)

$p = 1, \dots, P$ part order

$t = 1, \dots, T$ tool type

Variables

*a. variables for flexible manufacturing systems (FMS) or FMS
sub-system in a hybrid system*

H = makespan (throughput time)

V_{tk} = number of tools of tool type t assigned to machining center
 k

$\bar{X}_i = \begin{cases} 1, & \text{if part type } i \text{ is selected for FMS sub-system} \\ & \text{in a hybrid system} \\ 0, & \text{otherwise} \end{cases}$

\bar{X}_{ijtk} = proportion of operation j of part type i to be processed
using tool t on machining center k .

$X_p = \begin{cases} 1, & \text{if part order } p \text{ is selected for FMS} \\ 0, & \text{otherwise} \end{cases}$

X_{pjtk} = proportion of operation j of part order p to be processed using tool t on machining center k

$$Y_{tk} = \begin{cases} 1, & \text{if tool } t \text{ is assigned to machining center } k \\ 0, & \text{otherwise} \end{cases}$$

b. variables for conventional manufacturing systems (CMS) or CMS sub-system in a hybrid system

V'_{ijk} = a 0-1 integer variable, if operation $j-1$ of part i is not allocated to machine k but operation j of part i is allocated to machine k , then $V'_{ijk} = 1$; otherwise, $V'_{ijk} = 0$.

X'_{ijkl} = number of components of part type i undergoing operation j assigned to machine k and to be transported to machine l ($j=0$ and $k=0$ indicate a dummy operation, i.e., the transportation from raw material storage; $l=K+1$ means a location other than the k locations where the K machines are located -- may be a final production storage place or a assembly department).

$$\bar{X}'_i = \begin{cases} 1, & \text{if part type } i \text{ is selected for CMS sub-system in a hybrid system} \\ 0, & \text{otherwise} \end{cases}$$

$$X'_p = \begin{cases} 1, & \text{if part order } p \text{ is selected for CMS sub-system in a hybrid system} \\ 0, & \text{otherwise} \end{cases}$$

X'_{pjk} = proportion of operation j of part order p assigned to machine (work center) k

Y'_{ijk} = a 0-1 integer variable; if operation j of part i is assigned to machine k , $Y'_{ijk}=1$; otherwise, 0.

Parameters

- a. *parameters for flexible manufacturing systems (FMS) or FMS sub-system in a hybrid system*

A	= planning horizon
A_k	= available machining time on machining center k , $A_k = A r_k$
ΔA_k	= remaining available processing time at machining center k
C_{ijtk}	= machining cost to process operation j of part order i using tool t on machining center k
CON_{tk}	= the contribution (in terms of W_p handled) of tool t on machining center k
K^f	= number of machining centers available in the FMS sub-system of a hybrid system
L_t	= tool life of tool type t
M	= a positive integer, indicating an upper bound on the number of operations that can be feasibly assigned to a tool-machine combination
Q_t	= number of slots needed by tool t
r_k	= maximum utilization limit of machining center k
\bar{t}_{ijtk}	= machining time needed to process operation j of part type i using tool t on machining center k
t_{pjtk}	= machining time needed to process operation j of part order p using tool t on machining center k
TQ_k	= number of slots available at machining center k
ΔTQ_k	= remaining slot space at machining center k
W_p	= weight of part order p

b. parameters for conventional manufacturing systems (CMS) or CMS sub-system in a hybrid system

b_i = material handling cost per unit part per unit distance

C = overall system running cost per unit time (except material handling equipment running cost)

C'_{ijk} = unit machining cost of operation j of part i at machine k ,
 $C'_{ijk} = t'_{ijk} C_k$

C_k = machining cost per unit time on machine k

d_{kl} = distance from machine k to l

D_i = the demand for part type i

H'_{ikl} = material handling cost per unit part i from machine k to l
 $(H'_{ikl} = b_i * d_{kl})$

HC_i = work in process inventory cost (per unit time) of all parts of type i

K^c = number of machines available in the CMS sub-system of a hybrid system

$K(i,j)$ = number of feasible machines (work centers) for job (i,j)

ΔL = the lead time difference between FMS and CMS sub-system

r = proportion of setup cost within total production cost

R_{ijk} = the cost per major setup for operation j of part i at machine k , $R_{ijk} = R'_{ijk} * R_k$

R'_{ijk} = the time needed per major setup for operation j of part i at machine k

R_k = major setup cost per unit time on machine k

S_{ijk} = the cost per minor setup for operation j of part i at machine k , $S_{ijk} = S'_{ijk} * S_k$

S'_{ijk} = the time needed per minor setup for operation j of part i at machine k

S_k = minor setup cost per unit time on machine k

SC_i = average setup cost of operation j of part type i in CMS,

$$SC_i = \sum_{j=1}^{J_i} \sum_{k \in K_{ij}} R_{ijk} / K(i, j)$$

t'_{ijk} = the time needed to process one unit part type i of operation j at machine k

t'_{pjk} = machining time needed to process operation j of part order p at machine k

c. Lagrangian relaxation and computation related parameters

ϵ = the maximum acceptable percentage error

λ_n = a scalar in iteration n , $0 < \lambda_n \leq 2$

S_u^n = a positive scalar step size in iteration n

u_{tk}^n = Lagrangian multiplier in iteration n

Z^* = the best (highest) lower bound (on the objective value of the Lagrangian problem) obtained so far

Z_1^* = the lower bound (on the objective value of the Lagrangian problem) obtained in the current iteration

ZD = the best (lowest) upper bound (on the objective value of the Lagrangian problem) obtained so far

ZD_1^n = the upper bound (on the objective value of the Lagrangian problem) obtained in the current iteration

Sets

- τ_k = a subset of tools assigned to machining center k
- ϕ_{tk} = a subset of part orders whose operations have been entirely or partially loaded to FMS
- I_1 = selected part type set, each of the part types has at least one part order having been selected
- I' = a set of part types allocated to CMS sub-system
- K_c = a set of machines available in the CMS sub-system of a hybrid system
- K_f = a set of machining centers in the FMS sub-system of a hybrid system.
- K_{ij} = a set of machines (work centers) in CMS sub-system of a hybrid system which can perform job (i,j).
- S_1 = a subset of (t,k) combinations for which the tooling requirement is less than the available tooling assigned
- S_2 = a subset of (t,k) combinations for which the tooling requirement equals the available tooling assigned
- S_3 = a subset of (t,k) combinations for which the tooling requirement is greater than the available tooling assigned

CHAPTER I

INTRODUCTION

1.1. Background

The main characteristics of modern manufacturing industry include

- . Variable batch size
- . Broad but less predictable product variety
- . Short lead time, and
- . High productivity requirement.

Conventional manufacturing systems, either the highly productive but inflexible transfer line or the flexible but less efficient job shop, can not properly reflect these characteristics. To handle this situation, an effective manufacturing structure, flexible manufacturing system (FMS), which combines the benefits of both transfer line and job shop, i.e., high productivity and flexibility, is developed.

An FMS can be defined as " an integrated, computer-controlled complex of automated material handling devices and numerically controlled machine tools that can simultaneously process medium sized volumes of a variety of part types." (Stecke, 1983). More elaborately, a typical FMS has several machining centers. The individual machining centers are quite versatile and capable of performing many different operations. The tools required for the operations are either entirely stored in tool magazines distributed among the machining centers or partially assigned to the magazines, depending on the capacities of

the magazine and the availability of tool transportation devices. Each machining center is generally equipped with automatic tool and part change devices which operate rapidly with virtually no setup time between operations. The transportation of parts, tools, fixtures and pallets is coordinated through a mix of AGVs, Robots and/or conveyor systems. The well linked transportation systems among the machining centers, load/unload stations and storages are controlled by one or more computers.

The most important advantages that an FMS can offer are high productivity and flexibility. Benefits which can be expected from the new production technology or the expensive investment if properly designed include(Ranky,1983; Bessant, 1986)

- . High output
- . Low input, in terms of unit manufacturing cost
- . Short lead time
- . Low work in process
- . High machine utilization
- . Reduced workforce requirements
- . Higher and consistent quality levels

FMS is a relatively new production technology and also represents expensive investment. According to the survey of 18 US FMSs conducted by Kochan (1985), the average cost is \$12 million. In UK, the survey of 30 UK companies by Bessant (1986) shows the value is £2.4 million. (including some flexible manufacturing cells). However, as Hughes (1987) said, "The best machinery made can't solve production problems

if it is wrongly applied". Whether or not an existing FMS can solve production problems, i.e., achieve the expected benefits against its heavy investment, depends to a great extent on how the planning decisions, or pre-release decisions (Hwang and Shogan, 1989), are made.

1.2. Purpose

The FMS planning problems are defined as decisions that have to be made before the FMS can begin to produce parts (Stecke, 1984).

The planning problems in FMS are different from and more difficult to deal with than in transfer lines and job shops since

- . each machine can perform many operations
- . each operation of a same type of parts may be processed at more than one machine
- . the system can process several types of parts simultaneously
- . large number of tools are used; the tool selection and allocation decisions and other related tooling strategies are to be determined before production

Due to these difficulties and other considerations, the FMS planning problem in its general form is hard to tackle directly. Stecke (1983) suggested a framework for dealing with the broad FMS planning problem, which includes the following issues:

- (1) Part type selection
- (2) Machine grouping
- (3) Production ratio

(4) Resource allocation, and

(5) Loading

However, as will be discussed in Chapter 2, the terminology 'part type selection' may not properly express the meaning of the issue of selecting parts before production. Therefore in this research the term 'part selection' is used instead of 'part type selection'.

This dissertation aims at solving the combined part selection (PS) and machine loading (ML) problems, which may be considered as a kind of decision integration at the planning level.

Part selection and machine loading decisions are two different FMS planning problems. As will be seen from the definitions of PS and ML problems in the next chapter, there are common restrictions and even common system performance measures which link the two. The restrictions could be tool magazine capacity, job-tool-machine compatibility, available machining time (depending on the preset planning horizon), etc. The system performance criteria could be production output (throughput), production input (machining cost, throughput time or makespan), machine utilization, etc. The PS decision made without considering the above-mentioned restrictions and other operational details often encountered in ML stage may be infeasible as a part of input for later ML decision. Since the final machine loading optimality greatly depends on whether or not a proper subset of part-orders is selected, the PS decision should be made in such a way that the above mentioned common constraints and the

operational details considered in ML stage are taken into account. However, most of existing researches in this area treats part selection and machine loading problems separately or sequentially (The detail literature survey is given in the next chapter). The two sets of individually obtained solutions may conflict with each other. Even if they are sequentially solved as suggested by Stecké (1983), considerable time and effort are needed in the iterative procedure to make the solutions feasible. To avoid these possible conflicts and unnecessary effort, a joint consideration of the two problems is strongly desirable.

In this research the combined PS and ML problem in three classes of FMSs: FMS I, FMS II and hybrid systems will be investigated. The three classes of FMSs will be described in Chapter 2.

The purpose of the dissertation is to

- (1) suggest approaches which integrate the PS and ML problems in FMSs I and II.
- (2) suggest an approach which concurrently handle load sharing, PS and ML problems in hybrid systems.
- (3) formulate analytical models to
 - .simultaneously solve PS and ML problems (and load sharing problem -- for hybrid systems) such that a set of consistent solutions can be obtained, and
 - .make better use of available high productivity and flexibility by considering both output and input measures in the three classes of FMSs.

- (4) develop efficient solution procedures to solve relatively large sized problems for the developed models.
- (5) investigate the effects of magazine capacity and available machining time on system performance in FMSs and the effect of batch splitting on some system input measure through ML problem in conventional manufacturing system.
- (6) demonstrate the computational implementation of the solution procedure by solving a representative model.

1.3. Outline of the Dissertation

The dissertation is presented as follows. Chapter 2 gives an overview of PS and ML problems. The existing research in this area is reviewed followed by a preliminary discussion of the framework of the methodology, concepts, definitions and other issues related to later chapters. Chapters 3 and 4 tackle the combined PS and ML problem in FMS I and II. The problem in FMS I is formulated as mixed 0-1 integer problems for both primary and secondary objectives. For FMS II the problem is modelled as mixed general integer problem for both primary and secondary performance measures. Illustrative examples are given. Associated solution procedures are suggested for the models. The PS and ML problem for hybrid systems is handled in Chapters 5 and 6. In Chapter 5 the load sharing between the FMS and CMS subsystems is also included. Chapter 6 mainly focuses on the loading problem in the conventional manufacturing subsystem or a 'pure' CMS. The demonstrative implementation of the solution algorithm is presented in Chapter 7. Conclusions are drawn in Chapter 8. Further research in this area is also suggested in Chapter 8.

CHAPTER II

PART SELECTION AND MACHINE LOADING PROBLEMS---AN OVERVIEW

2.1 Definitions

Before defining part selection and machine loading problems, several concepts have to be introduced. Some of the concepts are adopted from Hwang and Shanthikumar (1987).

a. Part Order

A part-order is an order of a type of part and can be defined as a multi-attribute tuple. The attributes generally include part type, order quantity, due date, price (dollar value of the part-order), etc. In reality, a part type may have more than one part-order.

b. Production Order

The production order consists of the entire set of part-orders.

c. Magazine Capacity

Magazine capacity of a machining center is the total number of tool slots at the machining center.

d. Planning Horizon

Planning horizon is a pre-specified time duration over which a sub-set of production orders is planned to be processed.

e. Available Machining Time

Available machining time at each machining center during a planning horizon depends on many factors, such as the planning horizon, expected breakdown rates of the machining center and the system, etc. The available machining time at each machining center is generally less than the planning horizon.

f. Job

In this research job is defined as an operation of a part order. A job may be performed at more than one machining center with different tools. Job is also a multi-attribute tuple. The attributes are part order and operation.

With the above mentioned concepts the part selection and machine loading problems can be defined.

a. Part selection Problem

Many previous researches (e.g., Stecké, 1983, 1984; Stecké and Kim, 1986, 1987; Kusiak, 1984) focus on the part type selection problem. The representative part type selection definition, as Stecké (1983) presented, is " From a set of part types that have production requirements, determine a subset for immediate and simultaneous processing". However, as discussed earlier, in practice, a part type may represent several part-orders. Each of the part-orders may have different due date requirements, production quantity, etc., and hence, may have different priorities in the selection decision. Therefore, for a same part type, some of its part-orders may be selected while some may be rejected depending on their priorities. This definition is

appropriate only when priorities of the part-orders of a part type are the same or when each part type has only one part-order.

Hwang and Shogan (1989) extend part type selection to part selection problem, which considers not only part type but also other attributes such as due date and production quantity. The associated definition is "Given a production order and a set of tools needed to process this order, how do we form the batches such that the part orders in each batch can be simultaneously processed by the system and the selected performance measures are optimized". This definition avoided the problem present in Stecke's definition. But both definitions did not mention the technological and capacity restrictions to the part (or part type) selection problem. This raises two problems:

- a) The part selection problem may disappear. Technological restriction and capacity limitation are two conditions that make part selection decision indispensable. Part selection is necessary only when at least one of the conditions holds.

- b) The decision made based on these definitions may not be feasible for subsequent machine loading problems, since the part selection decision is an input to the machine loading problem.

To avoid the above-mentioned problems, in this research the part selection problem is defined as a problem of choosing a subset of the given production order for simultaneous processing on the specified machining centers using the specified tools during the following planning horizon such that some specified system performance measures can be optimized under technological and capacity constraints.

A full PS problem is a decision of selecting part orders with consideration of the compatibilities and capacities of both tools and machining centers. Here, the part order, the tool, and the machining center are three entities for the PS problem. Any selection problem without dealing with all of the three entities is considered as a partial PS problem in this research.

It is noted that from the above, the part selection problem is equivalent to the part type selection problem only when each part type has one part-order, or in other words, if each part type has only one part order the part selection problem reduces to a part type selection problem.

b. Machine Loading Problem

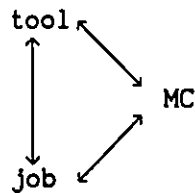
The machine loading problem in this research is defined as the decisions associated with allocating jobs (the operations of the selected part-orders) and required tools among the machining centers subject to technological and capacity restrictions such that the specified system performance measures are optimized.

As can be seen in the definition, the ML problem includes three entities: the job, the tool and the machining center. A machine loading problem taking care of all of the three entities is a full ML problem; otherwise it is called a partial ML problem.

One of the features of FMS is that each machining center can perform

many jobs. In reality, a job could be processed on several alternative machines using several alternative tools. To make better use of a FMS, each job should be allowed to be processed on alternative machining centers using alternative tools and each type of tool should be allowed to be allocated to alternative machines. If this is the case, the relationship between job and machine, or between job and tool, or between tool and machine is a many-to-many relationship. In contrast, if a machining center can perform many jobs but each job can be assigned to only one machining center, it is a many-to-one relationship. Similar many-to-one relationship may be assumed for job-tool or tool-machine relationship.

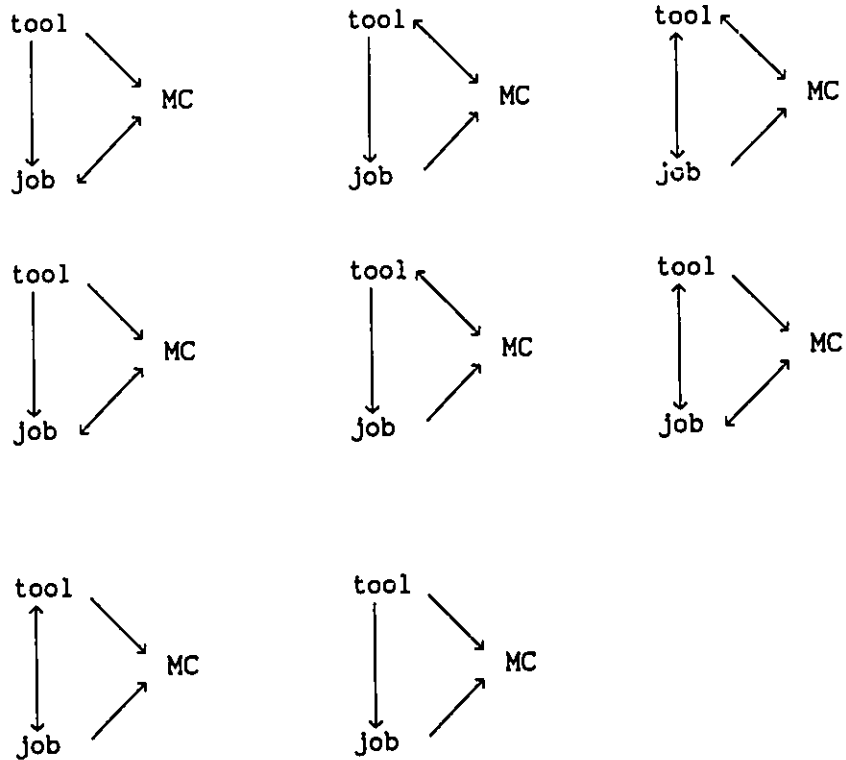
The illustrations of various ML problems are shown in Figures 2.1.a and 2.1.b.



Note: (1) \longleftrightarrow indicates a many-to-many relationship

(2) MC stands for machining center

Figure 2.1.a. Full machine loading problem with
many-to-many relationships



Note: \longrightarrow indicates a many-to-one relationship

Figure 2.1.b. Full machine loading problems with some types of many-to-one relationships

c. The Combined PS and ML Problem

The combined PS and ML problem considers the PS and the ML problems simultaneously. The combined PS and ML problem includes four entities: part order, job, tool and machining center. The combined PS and ML problems can be depicted in Figures 2.2.a,b and c, where, 'part order

\longleftrightarrow job' indicates the jobs of the selected part orders and the possibility of being selected of the part orders in turn depends on the adaptability of their jobs to the given tools and machines; 'part order \longleftrightarrow tool' and 'part order \longleftrightarrow MC' means part selection is performed under tooling and machine related constraints.

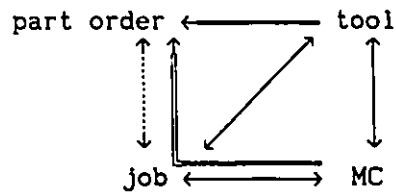


Figure 2.2.a. Full combined PS and ML problem

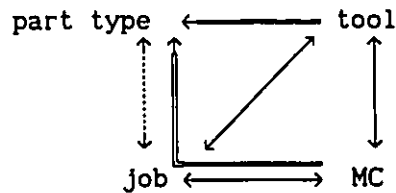


Figure 2.2.b. Partial combined PS and ML problem

In stead of considering alternative machines and tools for each job, some researchers, e.g., O'Grady and Menon (1987), consider alternative routes for each part order. In this case, a full combined PS and ML problem may be illustrated as shown in Figure 2.2.c.

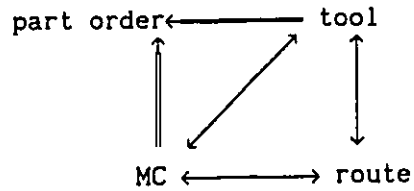


Figure 2.2.c. Another form of full combined PS and ML problem

This research focuses on the full combined PS and ML problem, the result can be easily extended to other types of PS and ML problems.

2.2 Review of Related Literature

2.2.1 Part Selection Problem

The part selection problem has not been extensively exploited. The limited literature on part selection problems can be classified into two approaches: flexible approach and batching approach.

2.2.1.1 Flexible Approach

This approach was suggested by Stecke and Kim (1986). The main idea is to improve system utilization by freeing up the space in the tool magazine whenever the the production requirements of some part type(s) are finished and probably introducing some new part types into the system for immediate and simultaneous processing. By simulation they demonstrated that this approach could achieve higher system utilization. But they considered only a flow shop type system. For a more complex case, e.g., a job shop, this approach may be very difficult to implement. In addition, some critical restrictions in the part selection problem, such as tool magazine capacity, were ignored.

2.2.1.2 Batching Approach

Batching approach attempts to partition the part types (part orders) into different batches; each batch is processed in a particular planning period or horizon. All of the selected part types (part orders) will be processed simultaneously. There will be no new part type (part order) inserted into the system even if some of the parts may be finished prior to the end of the horizon. The tools will stay in the magazine where they have been allocated at the beginning of the period until all the part types (orders) are finished.

This approach includes two methods: largest weightage method and similarity measures.

a. The Largest Weightage Method

Whitney and Suri (1984) reported two computer programs for part and machine selection. The first program selects both machines and parts based on the criterion of total dollar value difference between the saving from parts and the cost for FMS machines. In this program, since some details such as sharing of tools among parts are neglected, the part selection decision made in this program is merely a 'first cut'. The second program considers only the part selection problem, where parts are selected based on their weights, or performance indices--so-called relative saving ratio.

Starting from the work of Hwang (1986), Hwang and Shogan (1989) proposed two part selection models. The first model is for part type

selection and the second one for part order selection. The part order is defined as a multi-attribute tuple. The attributes are the part type, the order quantity, and due date. The due date consideration is taken care of by the weight, a function of due date, associated to each part order. The tool magazine capacity constraints is also considered in both models. The available machining time constraint appears in the second model where part order quantity is incorporated. According to Hwang and Shogan (1989), the production time is partitioned into machining time and set-up time. The issue of minimizing machining time is related to FMS loading problem and, therefore, should be investigated at the machine loading decision stage. In contrast, the setup time is associated with the part selection problem. By assuming that the time for each setup is constant, they concluded that the objective is to minimize the number of setups.

However, since this part selection formulation is intractable, they convert this problem into maximizing of part types or weighted part orders in each batch. An integer programming approach was applied to formulate this problem. In their model, the important restrictions, such as, magazine capacity, available machining time and part-tool allocation feasibility are considered. A Lagrangian relaxation based solution procedure was adopted which can efficiently solve real-world part selection problems.

The limitation of the models is that they can be only applied for some 'advanced' FMSs where all machines are identical and are general

purpose machines, i.e., each machine can perform all required operations and, therefore, implicitly assume an unlimited versatility. The whole system is treated as a single machining center and all tool magazines are treated as one 'big' unit. This may create another problem: the solution obtained based on this assumption may not be feasible or, at least, the real tool magazine capacity is actually inflated. This is because in the models, when several parts need the same tool only one tool is assigned to one central magazine. However, in a real situation, all magazines are located at different machines. At the same time, different parts with identical tool requirements may be allocated to different machines, requiring each machining center to have a this type of tool and, hence, indicating higher tool slot requirement than in the case when the magazines are centralized. This shows the infeasibility of the 'central' magazine manipulation.

b. Similarity Measures

This method emphasizes the selection of similar part types which require similar design and manufacturing processes. Several similarity measures or indices have been suggested.

Whitney and Gaul (1984) aimed at minimizing the number of distinct batches and tried to sequentially balance workload and improve tool sharing.

Another similarity method is proposed by Kusiak (1984), which selects parts based on a 'distance' measure. The objective is to choose a set of parts for simultaneous production such that the total 'distance' is

minimized. Kusiak (1985c) has also formulated the problem as a p-median problem to minimize the sum of the 'distances' from each part in a part family to the part family median. The problem was solved using subgradient method.

Gongaware and Ham (1981), and Han and Ham (1986) applied similarity vectors and part codes to formulate part families using clustering techniques.

The common problems with the methods using similarity measures, as Hwang and Shogan (1989) indicate, are

- a) the constraints, such as magazine capacity, available machining time, are ignored. This in turn results in the solution being infeasible.

- b) the similarity measures are often subjective and the meaning of the functional values of these indices is not clear.

2.2.2 Machine Loading Problem

The machine loading problem has attracted a number of researchers. Based on the tools applied, the research in this area may be classified into two categories: simulation approach and mathematical programming approach.

2.2.2.1 Simulation Approach

The available literature on this approach includes the work conducted by Steckel and Solberg (1981). In their study, an experimental investigation using simulation methods on different operating

strategies for a real FMS has been reported. The operating strategies considered involve policies for loading and real time flow control. The simulation results show that system performance depends significantly on the loading and control strategies chosen to operate this FMS. The simulation study also indicates that some degrees of pooling or duplicate operation assignments, i.e., allowing one operation to be assigned to more than one machine, would produce measurable improvement of system performance.

2.2.2.2 Mathematical Programming Approach

Most of the machine loading research is along this line. Since most of the models developed are quite complex and hard to solve some researchers may also turn to heuristics to find approximate solutions.

Kusiak (1983) presented four linear integer programming models for machine loading problems using the same objective, i.e., minimization of total processing cost, with different constraints. Simple linear integer structure and some practical constraints like tool life are the major features of the set of models. Several limitations, however, exist, such as

- a) two of the models have no consideration of tool magazine capacity restriction;
- b) another two models include tool magazine capacity considerations but no duplicate tool assignments allowed, which results in considerable slot space waste and unnecessary tool assignments.

Other drawbacks of these models, as summarized by Sarin and Chen (1987), include the following

- a) the operations of a batch of jobs are assumed uniform and continuously divisible, i.e., a portion of a batch can be arbitrarily assigned to each work station with different unit cost;
- b) every operation of a batch is assumed to have identical processing time.

Six separate non-linear integer programming models, each corresponding to a loading objective, were developed by Stecké (1983). All the models have similar constraints: the limitation on the number of duplicate operation assignments and the tool magazine capacity. The tool slots saving from the overlap are also considered. The problems with these models are

- a) optimization of one objective may lead to some negative effects on other objectives since the six objectives are separately considered;
- b) tool life is not considered;
- c) the solution procedure is very complicated due to the non-linear feature of the models; even when the linearization step is applied the procedure is still quite tedious. Later, a branch and bound based procedure was suggested by Berrada and Stecké (1984) to avoid the linearization step and solve the problem directly.

Another 0-1 integer model was developed by Co (1984) to maximize total machine flexibility, minimize the number of consecutive operations to be processed on the same machine, but computational implementation of

the model was not suggested.

Following Stecké (1983), Shanker and Tzen (1985) proposed a non-linear integer formulation, subject to similar constraints that Stecké (1983) applied, to minimize the system workload unbalance and the number of late jobs. Because of its non-linear structure, the model is difficult to solve directly or even by linearization. Therefore, some heuristics were suggested. Different dispatching rules were also investigated by simulation. In this model the tool allocation problem was not directly considered. The paper specified the number of slots required for operation of a job on a machine. This creates another problem, i.e., it is not clear how this could be done without knowing which tool would be used to perform this operation, since a job could be processed using different tools with different slot space requirements.

To investigate the machine loading problem for flexible assembly, Ammons *et al.* (1985) formulated a bicriterion model. The objectives are to balance work station workload and to minimize the total number of job-to-work station assignments. Since this model does not have an encouraging structure, heuristic methods were developed for large sized problems. It should be noted that in the case of flexible assembly, the tooling problem is insignificant and consequently, the solution procedure is relatively easier.

Chung (1986) formulated three objective functions: minimization of the difference in the total processing times between any two machines;

minimization of total processing time and minimization of total number of part-to-machine visits subject to tool magazine capacity and unique operation assignment (each operation can be assigned to only one machine). But he did not mention how the two constraints are linked to the third objective. No direct solution procedure was given. Two heuristics were proposed but no application example was illustrated. In addition, the tool assignment problem was not taken into account.

Sarin and Chen (1987) proposed more detailed 0-1 integer linear programming model which considered most of the important constraints, such as tool lives, available machining time and job-tool assignment feasibility. The objective was to minimize total machining cost. As opposed to other models, this model suggested different cost and processing times for different machine-tool-operation combinations. Efforts were also made to investigate the effects of lower utilization limit, tool-operation assignment limitation, tool life, and tool magazine capacity. Computational refinements of the model based on Lagrangian relaxation were also suggested.

Lashkari *et al.* (1987) investigated operation allocation problem which was formulated as a 0-1 nonlinear problem. Two objectives: minimization of transport load and minimization of refixturing activities, were separately considered. To solve the nonlinear programming model a linearization strategy was suggested. Associated computational experience was also reported.

A tool loading method, formulated as a non-linear 0-1 integer model,

for a special type of FMS, where each machining center can perform any operation on any part if appropriate tools are provided and a part is processed by only one machining center, was presented by Han, *et al.* (1989). Further assumptions include a) all machining centers are identical; b) each tool requires equal space in the tool magazine. Because of the unique feature of the FMS considered in this case and the assumptions, the operation assignment problem disappears and the machine loading problem reduces to a tool loading problem. The performance criterion is the amount of tool traffic among the machining centers and between the machining centers and the tool crib. A heuristic solution method was suggested. Some simulation experiments were also reported. It is clear that the application of this method is quite limited due to some of assumptions and the special type of the FMS considered in the paper.

For the same problem as stated in Sarin and Chen (1987), Ram, *et al.* (1990) developed a discrete generalized network model with simple side constraints, with the objective of minimizing total machining cost. Most of the important constraints such as tool life, magazine size, and available machining time were included in the model. An algorithm for solving the model was proposed. One of the features of the model is its ease of application to other FMS planning problems. Certain negative effects on system performance, however, may be resulted from some of the restrictions imposed in this model, such as

- a) each type of tool can be assigned to only one machine; and
- b) each operation is allocated on only one machine and processed using only one type of tool.

Shanker and Srinivasulu (1989) proposed a two-stage branch and backtrack procedure and associated 0-1 integer linear programming models. They also developed heuristic procedures with a bicriterion objective of minimizing the workload imbalance and maximizing the throughput for the critical resources. The similar problems that Shanker and Tzen (1985) faced earlier surface here again, i.e.,

- a) tool assignment problem is not explicitly considered;
- b) the number of slots required for a particular operation on a particular machine is known and fixed but it is not clear how this was done without being aware of which type of tool would be used for this operation. Since the consideration that several operations may need some of the same tools is neglected, i.e., the nonlinear constraint proposed by Stecké (1983) to handle the slot savings was dropped here, tool slot wastage and redundant tool assignments may result.

Bretthauer and Venkataramanan (1990) presented an integer model to study the operation assignment problem. The objective is to maximize a weighted sum of the number of operations to machine assignments subject to magazine size, available tools and the budget for purchasing additional tools. But the financial or management meaning of the objective is not clear. It seems that the purpose of the model is to maximize the weighted sum of alternative machines for improving the number of weighted alternative machines itself rather than for any other meaningful production purpose. Therefore the decision made based on 'optimum' solution of the model may result in some important objective values such as throughput, machining cost, throughput time

and specifically material handling cost, being unacceptable, although the maximum weighted sum of alternative machines is reached. Another problem with this model is that tool assignments are not considered.

Chen and Askin (1990) compared the performance of six loading heuristics on three existing FMSs. The performances of the heuristics were investigated based on separate evaluation of the five objectives: workload balance, amount of inter machine part movement, routing flexibility, tool investment and machine utilization. They concluded that none of the heuristics would likely meet the needs of all managers. Decision makers have to determine the criteria of greatest importance to their own manufacturing environment in choosing a loading heuristic or procedure, i.e., the choice of the primary objective and heuristic may vary from one manufacturing environment to another.

2.2.3 The Combined PS and ML Problem

Enough effort has not been made to handle the combined PS and ML problem. The available literature on the subject includes the work of Chakravarty and Shtub (1984), Rajagopalan (1986), and O'Grady and Menon (1987).

In the paper of Chakravarty and Shtub (1984), each workpiece is identified by the subset of tools required for its machining; the tools and workpieces are first grouped using group technology. The workpieces are clustered into groups based on their similarities in terms of tool requirements. One or more groups of workpieces are

selected for simultaneous processing in the FMS as one batch. Two 0-1 mixed integer models were developed. The first model is a minimax problem which is for only conceptual purposes. Its solution is very cumbersome because of non-linearity. The second model is a 0-1 mixed linear integer model which is used to load the workpieces of a single group (which is predetermined using a grouping procedure) on the FMS such that the maximum processing time among all machines can be minimized. Several notable problems in this model are:

a) No alternative subsets of tools are considered. This may restricts the utilization of the flexibility of the FMS.

b) The workpiece is actually handled as a bi-attribute tuple. The two attributes are order quantity (represented by processing time) and part type which is represented (identified) by the subset of tools required for processing the part. Another important attribute, i.e., due date is not taken into account. This may result in some parts which have lower similarities with others not being processed on time even if their priorities in terms of due date requirements are higher.

c) The relationship between the two models is not clear. If the first model is solved the second one is not needed to be solved since all decision variables have been fixed from the solution of the first model. If the first model is only for conceptual purposes and only the second model is used, the constraint associated with the magazine capacity on each machine is ignored.

d) The assumption that each tool requires only one tool slot is not practical.

e) Tool life is not considered.

a) and b) indicate that their work is a partial combined PS and ML

problem.

Rajagopalan (1986) proposed a formulation and associated heuristic solutions for parts grouping and tool loading problems which partially dealt with the combined PS and ML problem. In the process, certain restrictive assumptions were made, e.g., a part-type either has only one operation or all its operations on a certain machine type are assigned to the same machine.

O'Grady and Menon (1987) presented a multiple criterion approach to choose the compatible subset of candidate orders. The multiple criteria are represented by weighted values of deviations from desired goals. As indicated by Gray, et al. (1988), however, it is not clear how the appropriate weights for each of the criteria have been determined. Another problem is that, in their model and the FMS studied, each machine is designated for a particular function (operation) or a specified process route. This obviously restricts effective utilization of the inherent flexibility of an FMS.

2.2.4 The Limitations of Existing Literature and Motivation for This Research

Based on the above literature review, at least the following limitations exist in the previous researches:

- 1) Most of previous researches focus either on the part selection problem or on the machine loading problem. The linkages represented by available production resources and management goals between the two are often ignored. Therefore, the conflicts or inconsistencies between

the two separately obtained solutions will be inevitable. Even for the 'pure' part selection or 'pure' machine loading researches, several problems are noticeable:

a) Part selection related researches

- . many researches consider only partial PS problems.
- . the studied FMSs are often strongly restrictive, e.g., flow shop type FMS (Stecke and Kim, 1986), 'advanced' FMS where all machines have unlimited versatility and identical capacities (Hwang and Shogan, 1989), etc.

. in most researches using similarity methods (Whitney and Gaul 1984, Chakravarty and Shtub 1984, Kusiak 1983, 1984) constraints present in the part selection problem are generally ignored. Also, the similarity measures are often subjective.

b) Machine loading related researches

- . many researchers consider only partial ML problems or full ML problems with some many-to-one relationship.

. the tool assignment problem is not explicitly addressed in some literature (Shanker and Tzen 1985; Shanker and Srinivasulu 1989), where tooling is indirectly considered through the number of slots needed by each operation. Two problems arising from this manipulation are: the advantage provided by the fact that one operation may be processed by a number of alternative tools has not been exploited, and slot waste and redundant tool assignments may result.

. in some researches where tool assignment problems have been considered, two problems exist: to gain computational simplicity, the price of slot waste and unnecessary tool assignments has to be paid (Kusiak, 1983); and to eliminate redundant tool assignments and slots

waste a very complex model structure (Stecke, 1983) was formulated and hence the price of excessive computation has to be paid.

- . highly restricted FMS (Han et al. 1989)

- . most researches impose a restriction that one operation can be assigned to only one machine and/or a specific type of tool can be assigned to only one machine

2) The limited studies which jointly considered PS and ML problems have the following limitations:

- a) dealing with the partial combined PS and ML problem: either a part-type has only one operation or all its operations are restricted to the same machine (Rajagopalan, 1986); considering only part type selection (Chakravarty and Shtub, 1984).

- b) dealing with the full combined PS and ML problem with some many-to-one relationship: each machine is designated for a particular operation or a particular process route (O'Grady and Menon, 1987) which seriously restricts effective utilization of the inherent flexibility an FMS can offer (O'Grady and Menon, 1987).

In summary, the following issues exist in the previous researches and need to be addressed, i.e.:

- . the studied FMSs are too restrictive

- . either tooling is not explicitly considered resulting in slot waste and redundant tool assignments, or tooling is considered but computation is intractable.

- . the critical production resources are not considered.

- . an operation can be assigned to only one machine and/or one type

of tool can be assigned to only one machine.

. a machining center is designated to only one operation or one route.

The motivation for this research is, therefore, to develop an integrated approach to simultaneously handle PS and ML problems so that a set of consistent solutions can be obtained while some of the problems present in the previous researches can be avoided.

2.3 Preliminary Remarks on the Proposed Research

2.3.1. Scope of Research

Before specifying the research scope, it is necessary to classify FMSs. FMSs can be classified in many ways. Groover (1980) and Shanker and Tzen (1985) divided FMSs into two types:

- 1) Dedicated FMS;
- 2) Random FMS.

The dedicated FMS is designed to produce a rather small family of parts with well-defined and limited processing requirements. The random FMS, on the other hand, is designed to machine a large family of parts with a broad range of features. In random FMS, the product mix is not completely defined when installing the system.

Browne *et al.* (1984) defined four types of FMSs:

- a) Flexible machining cell;
- b) Flexible machining systems;

- c) Flexible transfer line;
- d) Flexible transfer multi-line.

A flexible machining cell consists of only one general-purpose CNC machine and automated material handling facility. A flexible machining system consists of several FMCs of different types of general-purpose machine tools and various material handling systems. This type of FMS allows several routes for parts, each with relative low production requirement. The third type of FMS is made up of both general and special purpose machines. The material handling system is usually a carousel conveyor or automated guided vehicular systems. For all parts, in this type of FMS, each operation is assigned to and performed on only one machine. This leads to a fixed route for each part. The flexible transfer multi-line consists of multiple flexible transfer lines.

This dissertation focuses on the random FMS which is type 2) according to Groover's (1980) classification and roughly correspond to type b) according to Browne *et al.* (1984). The PS and ML problems in random FMS are more difficult than in other types of FMS. Also, the methodologies applied and the solution procedures for the PS and ML problems in random FMS can be extended to other types of FMS.

However, random FMS represents a wide range of FMSs and therefore can be further divided into several classes. Since "tooling creates one of the most serious problems in FMS planning and application" (Tomek, 1986), random FMSs are classified based on tooling strategies.

(Henceforth the term FMS will be used to indicate random FMS).

In accordance with the suggestions by Tomek (1986), two types of FMSs corresponding to two tooling strategies are defined in this dissertation:

. *FMS I*

In this type of FMS, the majority of tool inventory creates a common tool storage, which is spread among machining centers. Tool transportation facilities are available which enable a sharing of tools among machining centers. Since tool transportation devices are available smaller tool magazine would be sufficient (ElMaraghy 1985) which in turn could offset the expenses of tool transportation facilities.

For a planning period, to minimize the tool movements among machining centers, the tools on the FMS should be preloaded.

. *FMS II*

Contrary to the tooling strategy described above, in this type of FMS each machining center has a bigger tool magazine (80-140 tools). The tool transportation facilities are not available or are unnecessary. Tool sharing among the machining centers is restricted in this case. Required tools are loaded onto magazines before production. Multiple spare tools of same type may be loaded on magazines depending on tool life and required machining time of this type of tool on a particular machining center.

The two types of tooling strategies have their own advantages and disadvantages. The tooling strategy in FMS I needs only a relatively small tool magazine which is less expensive and requires lower tool investment but this may be offset by the cost of tool transportation devices. FMS II requires higher tool investment and larger tool magazines which are more expensive; this does not need investment on tool transportation facilities. In terms of real time control, FMS I has to be supported by more powerful system software to control tool flow which should match the requirements of part (job) flow. In FMS II, on the other hand, tool management is much easier than in FMS I and less powerful software is sufficient.

. Hybrid FMS

In this type of FMS, conventional manufacturing sub-system (CMS), like job shop or other functional layout facilities, and flexible manufacturing sub-systems, e.g., flexible manufacturing cells, 'islands of automation', etc., exist simultaneously. The FMS sub-system could be either FMS I type or FMS II type. The parts which can not be processed within the FMS or can not make better use of FMS' advantages are processed in the CMS (Kusiak, 1987). This type of FMS exists because it "has given advantages in terms of cost and learning and allows small companies to legitimize investment over a number of years" (Bessant and Hayward, 1986).

This dissertation will concentrate on the combined PS and ML problems for these three types of FMSs.

2.3.2 A Framework of the Proposed Approach

To obtain a set of consistent solutions for the combined PS and ML problem, the decision variables related to part selection and machine loading (operation allocation and tool assignment) should be integrated into one model. Once this model is solved, a concurrent solution for both PS and ML decisions will be obtained.

The solution procedure developed in this research initially attempts to solve problems related to both PS and ML problems through the primary objective. If the decision maker is not satisfied with the resultant solution, then this can be refined further by introducing the secondary objective.

The framework of the proposed approach to solve the combined PS and ML problem in the three types of FMSs is illustrated in Figure 2.3. As shown in Figure 2.3, the approach is implemented in two sequential stages. The first stage corresponds to the primary objective. If no secondary objective is desired, the output from the first stage can also provide a concurrent, consistent solution for both part selection and machine loading problems. If the secondary objective is required then the second stage should be performed based on the results of first stage. The reason for doing so is that it is assumed that the primary objective is so important in comparison with any secondary objective that the secondary objectives should be considered only after the primary objective has been fully achieved under given constraints; in other words, the improvement of secondary objectives

should not have any negative effect on the primary objective. Therefore, it would be improper to simply maximize or minimize the weighted summation of the primary and secondary objectives, since in doing so the improvement of secondary objectives will sacrifice somewhat the primary objective; besides that, the weightages assigned to primary and secondary objectives are often fairly subjective and may not reflect the true levels of importance.

2.3.3 Selection of System Performance Measures

a. Primary Objective

For any system, maximizing system output is a natural primary objective given the fixed amount of input. For an FMS, the output could be throughput, the dollar value of the parts, other productivity measures, or satisfaction of due date. The input could be available processing time (depends on planning horizon), magazine capacity, flexibility of the FMS, etc. Therefore the primary objective is to select and load a subset of parts such that the system output or, alternatively, the FMS productivity, can be maximized under the restriction of production resources.

b. Secondary Objectives

Once the maximum system output has been achieved, the next consideration is to more economically process the selected parts, or to obtain the maximum system output with less input. In our problem, this means either minimizing processing cost or reducing makespan, given that the primary objective has been fulfilled. The secondary

objective problem arises from two facts: a) alternative ways exist for both job and tool allocations so that the secondary objective(s) can be improved by rearranging the job and/or tool allocation pattern; and b) the planning horizon is set before production; the available processing time constraint may be much less restrictive than the magazine capacity constraint, which leads to considerable time redundancy. The reduction of makespan can be achieved by eliminating this time redundancy and rearranging job and tool allocations. Since the improvement in secondary objectives relies to a great extent on the flexibility of an FMS and the flexibility of the machining centers, improving secondary objectives actually enhances the capability to utilize the built-in flexibility, or the capability to convert flexibility to some economic benefits.

It should be pointed out that the alternative optimal solutions may exist for the primary objective model. Choosing different solutions from the alternative optimal solutions of the primary objective model may result in different secondary objective solutions. This indicates that a better chance exists to improve secondary objective. However, it is difficult to obtain the alternative optimal solutions for the primary objective. This issue is not investigated in this study but it may be an interesting topic for future research.

Some other measures may also be used. Here only these commonly applied measures are used. The following chapters will show how these measures are used in the combined PS and ML problems in different FMSs.

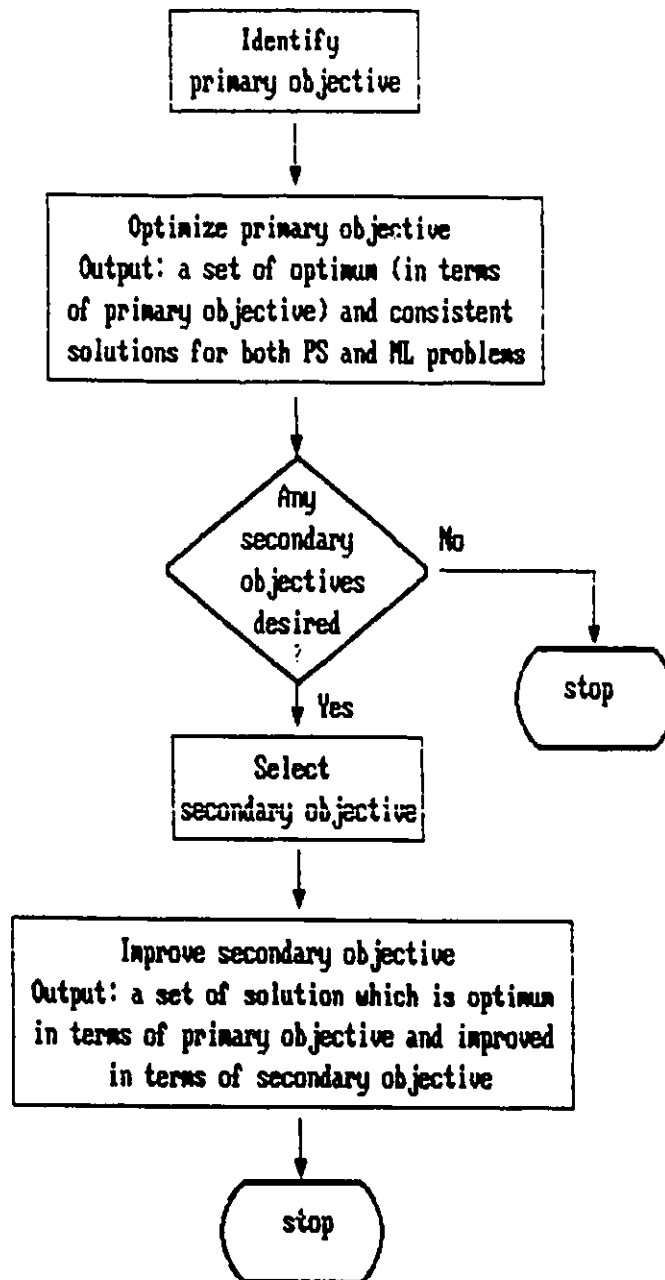


Figure 2.3 Structure of the integrated approach to part selection and machine loading

CHAPTER III

THE COMBINED PART SELECTION AND MACHINE LOADING PROBLEM IN FMS I

3.1 Problem Description

This type of FMS comes mainly from the tooling strategy that has been discussed in the previous chapter. The tooling strategy, in turn, results from the production pattern described below.

According to the survey of 22 FMSs in the U.S. conducted by Smith *et al.* (1986), 47% of the surveyed FMSs group parts into batches that are periodically rerun. Among them, 63% of the FMSs change batches on a daily base. In this type of manufacturing environment, the batch sizes are generally small. This may be supported by a survey performed by Bessant and Hayward (1986) which shows that 58% of the 50 FMSs (FMCs) of 30 Britain companies deal with batch sizes of 1-10, and 15% of them deal with batch sizes of 11-25. Similar figure is also available in Japanese companies. While reviewing several FMSs in Japan, Ito (1981) mentioned the batch size ranging from 6 to 30.

Because of the relatively small batch sizes and also because only one tool can be used at a time, it is unnecessary to assign any tool more than once to the same machine (Stecke, 1983). Similar arguments are also provided by Hwang and Shanthikumar (1987). Gray *et al.* (1988) also mentioned "only one copy of each cutter needs to be loaded, which saves some magazine capacity". Furthermore, in some FMSs, the tools are very expensive (e.g., some coated tools); it may be economically

prohibitive to assign more than one tool of the same type to a machine. The spare tools of the same type are available but in very limited numbers. Therefore, an assumption in this chapter is that only one tool of each tool type can be assigned to a machine where the tool is required. The validity of this assumption is also supported by the fact that assigning only one tool of each tool type to a machine can increase the effective magazine capacity and the machine flexibility (Gray et al. 1988). The possible tradeoff is that it may increase the number of times that a machine is stopped to change tools.

Consider an FMS of this type with K machining centers. Each machining center can perform a range of different operations. The available processing time (which depends on the given planning horizon) and magazine capacity are known and fixed. It is assumed that the tool magazine is unchangeable and is fixed to the machine (Sarin and Chen 1987). There are P candidate part orders to be selected for immediate and simultaneous processing. The order sizes of the P part orders are known and are fixed at the beginning of the period. Each part order needs a certain number of operations (J_p). Each operation of a part order may be processed at more than one machining center using similar or different types of tools. Here the commonly used assumption that each operation is assigned to only one machine and processed by only one type of tool has been dropped. The advantage of relaxing this assumption was investigated by Liang and Dutta (1990). The compatibility of each tool-machine and tool-job combination is known. It is also assumed that the data related to processing cost and time of each job-tool-machine combination are given. The operation sequence

of a part is assumed to be known and no alternative operation sequences are considered. Our purpose is to develop an integrated approach to solve the combined PS and ML problem in the above-mentioned FMS. The frame work of this approach is illustrated in Figure 2.3.

The following notations will be used in this chapter.

Subscripts

- $i = 1, \dots, I$ part type
- $j = 1, \dots, J_p$ operation
- $k, = 1, \dots, K$ machining center
- $p = 1, \dots, P$ part order
- $t = 1, \dots, T$ tool type

Variables

- H = makespan (throughput time)
- \bar{X}_{ijtk} = proportion of operation j of part type i to be processed using tool t on machining center k .

$$X_p = \begin{cases} 1, & \text{if part order } p \text{ is selected} \\ 0, & \text{otherwise} \end{cases}$$

- X_{pjtk} = proportion of operation j of part order p to be processed using tool t on machining center k

$$Y_{tk} = \begin{cases} 1, & \text{if tool } t \text{ is assigned to machining center } k \\ 0, & \text{otherwise} \end{cases}$$

Parameters

A	= planning horizon
A_k	= available machining time on machining center k , $A_k = A r_k$
ΔA_k	= remaining available processing time at machining center k
C_{ijtk}	= machining cost to process operation j of part order i using tool t on machining center k
CON_{tk}	= the contribution (in terms of W_p handled) of tool t on machining center k
M	= a positive integer, indicating an upper bound on the number of operations that can be feasibly assigned to a tool-machine combination
Q_t	= number of slots needed by tool t
r_k	= maximum utilization limit of machining center k
\bar{t}_{ijtk}	= machining time needed to process operation j of part type i using tool t on machining center k
t_{pjtk}	= machining time needed to process operation j of part order p using tool t on machining center k
TQ_k	= number of slots available at machining center k
ΔTQ_k	= remaining slot space at machining center k
W_p	= weight of part order p

Lagrangian relaxation and computation related parameters

ϵ	= the maximum acceptable percentage error
λ_n	= a scalar in iteration n , $0 < \lambda_n \leq 2$
S_u^n	= a positive scalar step size in iteration n

u_{tk}^n = Lagrangian multiplier in iteration n
 Z^* = the best (highest) lower bound (on the objective value of the Lagrangian problem) obtained so far
 Z_1^* = the lower bound (on the objective value of the Lagrangian problem) obtained in the current iteration
 ZD = the best (lowest) upper bound (on the objective value of the Lagrangian problem) obtained so far
 ZD_1^n = the upper bound (on the objective value of the Lagrangian problem) obtained in the current iteration

Sets

τ_k = a subset of tools assigned to machining center k
 ϕ_{tk} = a subset of part orders whose operations have been entirely or partially loaded to FMS
 I_1 = selected part type set, each of the part types has at least one part order having been selected

3.2 Model Formulation

A set of models is developed such that, once solved, the following decisions can be made:

- 1) part selection
- 2) job (operation or proportion of operation) allocation
- 3) tool selection and assignment.
- 4) process route selection

The primary and secondary objectives have been specified in chapter 2, namely, output measure (primary) and input measures (e.g., machining

cost or makespan--secondary).

Model for Primary Objective

Given the foregoing primary objective, an appropriate model should be formulated in such a way that the solution should give an answer to both part selection and loading (job assignment and tool allocation) decisions. Keeping this in mind, the combined PS and ML problem can be formulated as a mixed 0-1 integer program as follows:

Model (M-1)

$$\text{Max } \sum_{p=1}^P W_p X_p \quad (3-1)$$

Subject to

$$X_p - \sum_{t=1}^T \sum_{k=1}^K X_{p,jtk} = 0 \quad \forall p$$

$$j = 1, \dots, J_p \quad (3-2)$$

$$\sum_{p=1}^P \sum_{j=1}^{J_p} \sum_{t=1}^T t_{p,jtk} X_{p,jtk} \leq A_k \quad \forall k \quad (3-3)$$

$$\sum_{p=1}^P \sum_{j=1}^{J_p} X_{p,jtk} - M Y_{tk} \leq 0 \quad \forall t, k \quad (3-4)$$

$$\sum_{t=1}^T Q_t Y_{tk} \leq TQ_k \quad \forall k \quad (3-5)$$

$$X_p = 0, \text{ or } 1 \quad \forall p \quad (3-6)$$

$$Y_{tk} = 0, \text{ or } 1 \quad \forall t, k \quad (3-7)$$

$$0 \leq X_{p,jtk} \leq 1 \quad \forall p, t, k \quad (3-8)$$

$$j = 1, \dots, J_p$$

From this model, it can be seen that variable X_p takes care of the part selection problem. Tool allocation is handled by variable Y_{tk} ; variable X_{pjk} is used to deal with job assignments and link variables X_p and Y_{tk} . In the objective function, the weight may represent order size (in which case, the objective is to maximize productivity) of a part order, the dollar value of a part order, satisfaction of due date (which is a function of duration until part order is due as suggested by Hwang and Shogan 1989), etc., depending on the decision maker. The constraint (3-2) ensures that the total proportion of part order p processed at all machining centers using all feasible tools should be the same for all operations (either 1 or 0). Constraint (3-3) says that the available processing time on each machining center can not be exceeded. Constraint (3-4) assures that if a job is allocated at machining center k the required tool should also be assigned to the machining center. The magazine capacity restriction is represented by (3-5). Constraint (3-6) means that a part order is either entirely selected or totally rejected. The integrality of indicator variable Y_{tk} is imposed by constraint (3-7).

The tool magazine capacity is considered as a constraint in model M-1 because a) "The most critical limitation of FMS is usually the tool magazine capacity. It restricts the number of tools that can be mounted on the magazine and, hence, constrains the number of part types that can be processed simultaneously." (Hwang and Shogan 1989); and b) "Although there may be many orders for various part types in various quantities, usually they cannot all be produced at the same

time because there is not enough space in the tool magazines to hold all of the required cutters." (Gray *et al.* 1988).

In model M-1, X_p is defined as an integer variable because p represents part order while all parts within the same order generally have the same due date requirement. This indicates that a part order should be either totally selected or entirely rejected for processing during the following period. However, if the parts within an order are allowed to be processed in different periods, model M-1 can be easily modified, i.e., by relaxing X_p to $0 \leq X_p \leq 1$, to meet this requirement. The solution method presented later can also be used to solve this problem with minor modification which will be addressed later.

After solving model M-1, the PS and ML decisions can be made based on the solution. However, the chance to improve secondary objectives still exists because of either the possible alternative optimum solution, which physically means existing multiple ways of assigning jobs and allocating tools given that all X_p 's are fixed, or extra redundancy of available processing time, which implies the possibility of reducing production cost or makespan. It should be mentioned that the problem of improving the secondary objective is considered given that the PS decision has been made, i.e., a subset of part orders has been selected for simultaneous production. Some of the selected part orders may belong to the same part type. In this case, the part orders belonging to the same part type have the same importance in terms of the concerned secondary objective and, therefore, are combined together according to their part type to simplify the model and

solution procedure. The associated models are formulated as follows.

Model for Secondary Objective 1 (Minimize Machining Cost)

Model (M1-1)

$$\text{Min } \sum_{i \in I_1} \sum_{j=1}^{J_1} \sum_{t=1}^T \sum_{k=1}^K C_{ijtk} \bar{X}_{ijtk} \quad (3-9)$$

Subject to

$$\sum_{t=1}^T \sum_{k=1}^K \bar{X}_{ijtk} = 1 \quad \begin{matrix} i \in I_1 \\ j=1, \dots, J_1 \end{matrix} \quad (3-10)$$

$$\sum_{i \in I_1} \sum_{j=1}^{J_1} \sum_{t=1}^T \bar{t}_{ijtk} \bar{X}_{ijtk} \leq A_k \quad \forall k \quad (3-11)$$

$$\sum_{i \in I_1} \sum_{j=1}^{J_1} \bar{X}_{ijtk} - M Y_{tk} \leq 0 \quad \forall t, k \quad (3-12)$$

$$0 \leq \bar{X}_{ijtk} \leq 1 \quad \begin{matrix} i \in I_1, j = 1, \dots, J_1 \\ \forall t, k \end{matrix} \quad (3-13)$$

and (3-5), (3-7).

where, I_1 is a subset of part types; for any of the part types in the subset at least one part order has been selected.

Model for Secondary Objective 2 (Minimize Makespan)

Model (M1-2)

$$\text{Min } H \quad (3-14)$$

Subject to

$$\sum_{i \in I_1} \sum_{j=1}^J \sum_{t=1}^T \bar{t}_{ijtk} \bar{X}_{ijtk} \leq H r_k \quad \forall k \quad (3-15)$$

$$H \leq A \quad (3-16)$$

and (3-10), (3-12), (3-5) and (3-7).

The application of the coefficients C_{ijtk} , t_{pjtk} , and \bar{t}_{pjtk} in the above models is based on references by Sarin and Chen (1987), Ram *et al.* (1990), and Jain *et al.* (1991).

It should be mentioned that the significance of reducing makespan by using model M1-2 depends on the following cases:

Case 1.

Both magazine capacity and available machining time constraints are loose or inactive. This can be identified from the output of primary objective where the information on the remaining available machining time and remaining number of slots on each machine can be obtained. In this case, maximizing throughput may result in the system being in a state of considerable workload unbalance (Shanker and Srinivasulu, 1989). The makespan may be reduced by relocating some jobs from the machine(s) with high workload to machine(s) with low workload or by simply eliminating the redundant available machining time.

Case 2.

The magazine capacity constraint is tight (or active) but the

available machining time constraint is loose or inactive. The makespan in this case may be reduced using similar methods as mentioned in case 1.

Case 3.

The available machining time is tight but the magazine capacity constraint is loose. In this case the available machining time may not be totally exhausted because of the integrity of X_p . So, the reduction of makespan is also possible. But the amount of reduction of makespan is very limited. Therefore, it is suggested that if the available machining time is the 'bottleneck' constraint the makespan may not be considered as the secondary objective given that the throughput is the primary objective.

It should also be emphasized that this set of models is a planning tool which focuses on part selection and machine loading problems. The job sequencing issue is not considered. The job sequencing problem is generally considered at the scheduling stage. Similar manipulations have been applied by Rajagopalan (1986). In his paper, the objective is to minimize the completion time which is equal to the total processing time plus the tool changeover time. The total processing time is equal to the maximum processing time among the machines or the processing time on the bottleneck machine. Moreover, as suggested by Stecké (1984), 'More appropriate for an FMS might be a *real-time, on-time* scheduling policy, with scheduling decisions based on the actual state of the system'. However, it should be mentioned that the minimum makespan obtained from the secondary objective model may be

affected by different scheduling methods.

3.3. Illustrative Example

To illustrate the attributes of the models developed, an example, along the same lines as those used by Sarin and Chen (1987), Co, *et al.* (1990), Chen and Askin (1990), Avont and Wassenhove (1988), Shanker and Tzen (1985), and Shanker and Srinivasulu (1988), is presented as follows.

Example 3.1

Consider an FMS with three machining centers. The planning horizon is 125 hours. The maximum utilization limit of each machining center is 80%. This comes up as 100 hours of available processing time on each machining center. Each machining center has a magazine with seven tool slots. 15 types of tools and tool transportation devices are available. Six candidate part types (each has only one part order) are considered for selection, each with 3 operations. The order sizes of the six types of parts are: 20, 30, 50, 10, 30 and 10, respectively. The tool-job and tool-machine compatibilities are listed in Table 3.1. The machining cost and time data for each job-tool-machine combination are shown in Table 3.2. The objectives are:

- . Primary: to select as many parts as possible for immediate and simultaneous manufacture, i.e., to maximize throughput. In this case the weight is the size of each part order.

- . Secondary: reduce machining cost, or makespan as much as possible.

The problem includes:

50 0-1 integer variables

810 continuous variables

69 constraints

After eliminating infeasible tool-machine and tool-job combinations, the problem size reduces to:

26 0-1 integer variables

50 continuous variables

44 constraints

The example problem has been solved using the LINDO micro-computer package. Each run took 15--20 minutes on a IBM AT microcomputer for both primary and secondary objectives. The unnecessary tool assignments are deleted by checking $\sum_p \sum_j X_{pjtk}$ value: if $\sum_p \sum_j X_{pjtk} = 0$, then let $Y_{tk} = 0$. The outputs are summarized in Table 3.3.

As shown in Table 3.3, after maximizing the primary objective, both PS and ML decisions can be simultaneously made if no secondary objective is desired. If a secondary objective is desired, however, the solution obtained by solving the primary objective is used only for PS decision while the ML decision is made by solving the secondary objective problem.

It is noted that the route of a part is automatically obtained from the solution to the model, *vide* Table 3.3. For example, from the 'Job assignment' (the solution to primary objective model) column in Table 3.3, the route for part 2 is derived as follows: 78% of operation 1 of part 2 (J21) has been assigned to machining center 2 (M2) and 22% of

J21 to machining center 1 (M1); operation 2 of part 2 (J22) is entirely (100%) allocated to machining center 3 (M3); and operation 3 of part 2 (J23) is entirely (100%) assigned to machining center 2 (M2). Therefore the route for part 2 is: $\overset{M1}{\text{M3}}-\underset{M2}{\text{M2}}$. Similarly the routes for other parts can be derived.

Table 3.3 also shows that optimizing the secondary objective can considerably reduce machining cost (14.6%) or makespan (11.2%) while keeping the maximum primary objective unchanged. If more redundant processing time exists, the higher percentage of makespan reduction can be expected. In Table 3.3 it can be seen that the job and tool allocation patterns are different before and after optimizing the secondary objective. This demonstrates that the improvement of the secondary objective can be achieved by re-arranging job and tool allocation.

To investigate the effects of magazine capacity and available machining time on throughput, additional computations are performed with different input parameters. Figures 3.1 and 3.2 show the results (to simplify the discussion, it is assumed that the magazine sizes and available machining times at all machining centers are the same). Based on Figures 3.1 and 3.2, the following observations can be made:

- . Magazine capacity has a significant effect on part selection decision but this effect may be bounded by available machining time. In other words, for a given amount of machining time A, there is a critical magazine size at which the throughput reaches maximum. Any additional slots above the critical magazine size will become

redundant. Any magazine with a size less than the critical size will restrict the throughput value.

. Similarly, the part selection decision is also affected by available machining time but the effect may be bounded by magazine capacity. As shown in Figure 3.2, for a fixed magazine size TQ , there exists a critical amount of machining time at which the throughput reaches maximum. Any additional machining time over the critical amount will not help to gain any additional benefits; simultaneously, if the available machining time is less than the critical amount certain tool slots will become redundant.

Tool type		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Tool-job compati- bility	(1)															
	J11		1		1										1	
	J12	1	1											1		
	J13									1	1					1
	J21			1			1		1							
	J22					1		1								
	J23								1	1						
	J31	1		1								1				
	J32					1					1		1			
	J33						1	1							1	
	J41	1														
	J42			1							1					
	J43				1											
	J51		1													
	J52								1							
	J53						1	1								
	J61				1				1							
	J62									1						
	J63										1					
Tool-M/C compati- bility	MC1	1	1	1	1			1				1		1		
	MC2	1			1		1		1		1		1		1	
	MC3			1		1		1		1				1		1
Required slots		4	2	4	4	3	1	3	3	2	4	1	2	3	2	1
Tool life		3	3	3	3	3	3	3	3	3	3	3	3	3	3	3

(1) J11--Operation 1 of part type 1.

Table 3.1. Tool-job and tool-machine compatibility,
and tool life and slot requirement data

Job	Compatible tools	Machining cost(\$100)			Processing time(hrs)		
		MC1	MC2	MC3	MC1	MC2	MC3
J11	2	50			25		
	4	40	50		20	20	
	14		40			18	
J12	1	30	28		10	15	
	2	22			10		
	13	28		20	12		15
J13	9			25			10
	10		40			20	
	15			12			12
J21	3	45		55	30		25
	6		75			35	
	8		80			40	
J22	5			44			20
	7	30		28	18		20
J23	8		22			10	
	9			15			6
J31	1	70	80		45	40	
	3	75		80	45		40
	11	85			40		
J32	5			32			15
	10		44			20	
	12		42			20	
J33	6		66			32	
	7	80		56	25		30
	14		70			38	
J41	1	25	20		8	10	
J42	3	25		25	10		20
	10		20			9	
J43	4	15	18		12	12	
J51	2	23			10		
J52	8		40			30	
J53	6		40			40	
	7	90		80	45		50
J61	4	15	14		5	8	
	8		10			5	
J62	9			20			10
J63	10		28			15	

Table 3.2. Machining cost and time data

Pri. obj.	MC	Job assignment (%)	Tool assignment	Cost (\$)	Planning horizon (makespan) (hrs)
		(1)	(2)		
	M1	J11/100, J12/100, J21/22, J31/100, J51/100	T2, T3		
Max. through- put	M2	J21/78, J23/100, J33/90, J52/100	T6, T8	50,930	125 (125)
	M3	J13/100, J22/100, J32/100, J33/10, J53/100	T5, T7, T15		

Part types selected: 1, 2, 3, 5. Throughput: 130

Process routes: part 1: M2-M1-M3, part 2: M1-M3-M2, part 3: M1-M3-M2, part 5: M1-M2-M3

Sec. obj.	MC	Job assignment (%)	Tool assignment	Cost (\$)	makespan (hr)
	M1	J12/100, J21/100, J31/100, J51/100	T2, T3,		
Min. Cost	M2	J11/100, J23/100, J52/100, J53/100	T6, T8, T14	43,500	122
	M3	J13/100, J22/100, J32/100, J33/100	T5, T7, T15		

Process routes: part 1: M2-M1-M3, part 2: M1-M3-M2, part 3: M1-M3-M3, part 5: M1-M2-M2

	M1	J11/100, J12/100, J22/100, J33/100, J51/100, J53/2	T2, T7		
Min. makespan	M2	J32/100, J52/100, J53/98	T6, T8, T12	50,300	111
	M3	J13/100, J21/100, J23/100, J31/100	T3, T9		

Process routes: part 1: M1-M1-M3, part 2: M3-M1-M3, part 3: M3-M2-M1, part 5: M1-M2-M2

- (1) J11/100 means 100% of job J11.
(2) T2 means tool type 2.
(3) M2-M1-M3 means operation 1,2 and 3 are processed at machining centers M1,M2 and M3 respectively.

Table 3.3. Part selection and loading solution for example 3.1

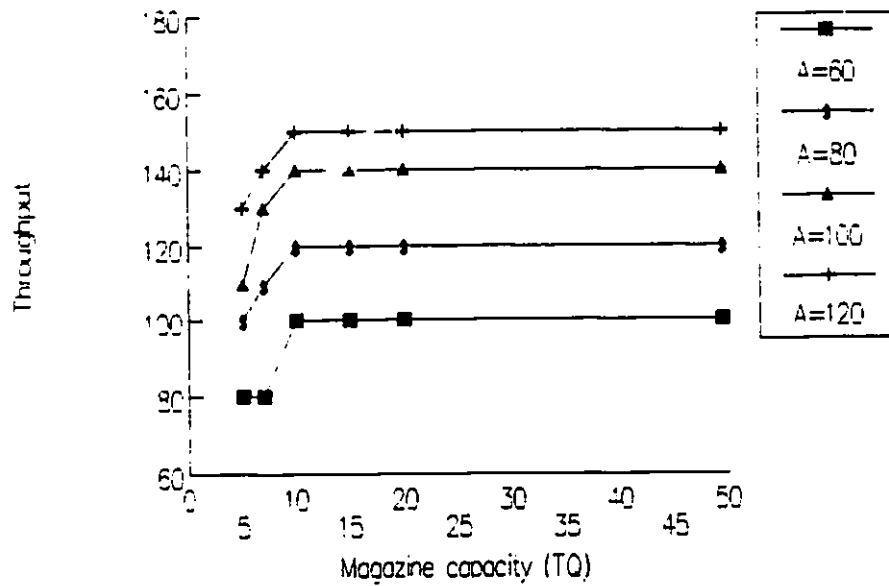


Figure 3.1 Effect of tool magazine capacity on throughput (for example 3.1)

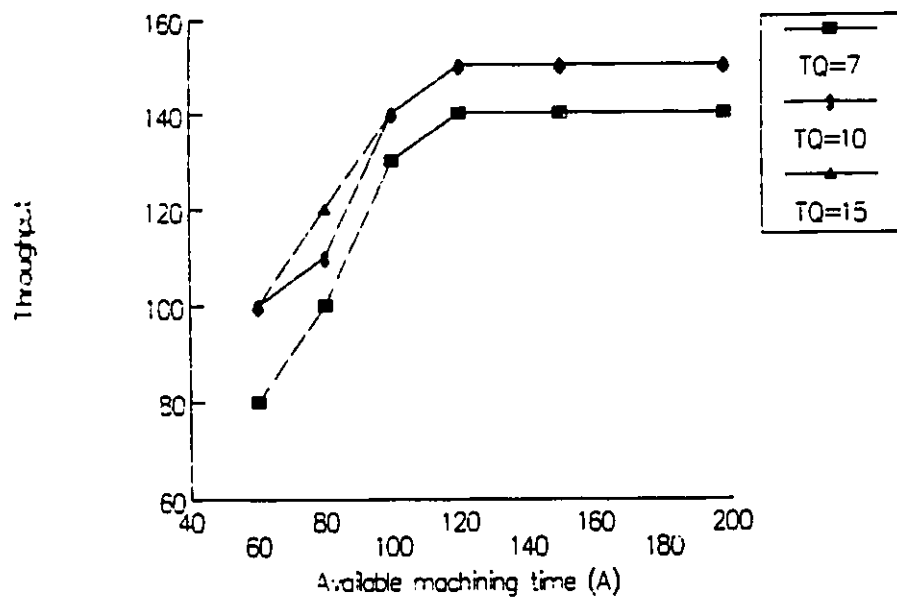


Figure 3.2 Effect of available machining time on throughput (for example 3.1)

3.4 Solution Methodology

3.4.1 Solution Procedure for Model M-1

The main difficulty in solving model (M-1) arises from the size of the mixed integer problem. To illustrate, consider an FMS with the following data: 30 part orders are to be selected; each part order requires 5 operations; 50 types of tools are available; 5 machining centers can be used. This involves 37,500 continuous variables, 280 integer variables and 410 constraints. In general, the problem size is (assuming $J_p = J$, for all p):

Number of integer variables = $P+TK$

Number of continuous variables = $PJTK$

Number of constraints = $JP+2K+TK$

A problem of this size is obviously hard to solve using existing commercial packages. Besides, part selection and machine loading are both low level production planning problems which are conducted weekly or daily--this means a quick solution procedure is required. Given this, we propose a solution method based on Lagrangian relaxation method incorporating decomposition principle (Bazaraa and Jarvis, 1977), which is implemented through application of the revised simplex method and a column generation scheme, for one of the subproblems resulting from the relaxation.

It should be mentioned that linear programming relaxation (LP relaxation) may also be used to solve the problem. The reasons for not using LP relaxation are:

1) The bounds obtained from Lagrangian relaxation (LR) are generally better than or at least as good as the ones obtained from LP relaxation (Fisher, 1981).

2) The model with LP relaxation (LP model) is more difficult to solve than the one with LR (LR model) since the LP model still includes a large number of variables and four sets of constraints with different structure which are difficult to be decomposed.

3) After solving the LP model more effort is required to make it feasible by rounding off X_p and Y_{tk} to integer values. However, for the LR model, only X_p should be rounded off to integers while Y_{tk} is already an integer from the solution to the 0-1 knapsack sub-problem.

a. Lagrangian Relaxation Procedure

For model (M-1), by relaxing the link constraint (3-4), the following Lagrangian problem is obtained

$$\text{LM-1:} \quad \text{Max} \sum_{p=1}^P W_p X_p - \sum_{p=1}^P \sum_{j=1}^J \sum_{t=1}^T \sum_{k=1}^K u_{tk} X_{pjtk} + \sum_{t=1}^T \sum_{k=1}^K M u_{tk} Y_{tk}$$

Subject to (3-2), (3-3) and (3-5)--(3-8)

This problem is equivalent to the following two independent sub-problems:

Sub1-1

$$\text{Min } \sum_{p=1}^P \sum_{j=1}^J \sum_{t=1}^T \sum_{k=1}^K u_{tk} X_{pjtk} - \sum_{p=1}^P W_p X_p$$

Subject to (3-2), (3-3), (3-6) and (3-8).

Sub1-2

$$\text{Max } \sum_{t=1}^T \sum_{k=1}^K M u_{tk} Y_{tk}$$

Subject to (3-5) and (3-7).

b. Solution Methods of sub1-1 and sub1-2

Subproblem sub1-2 consists of K independent single 0-1 knapsack problems. Each of the K single 0-1 knapsack problems can be easily solved by the algorithm developed by Martello and Toth (1978).

Sub1-1 is a large scale mixed integer programming problem. The difficulty involved in solving this subproblem lies in the large number of variables or columns, and the integrality of X_p 's. Since the main purpose of solving the relaxed problem is to obtain an upper bound for the original problem, a natural simplification is to relax the integrality imposed on X_p 's. In doing so an upper bound can still be obtained but the sacrifice is a tight upper bound. The X_p 's with values less than 1 will be deleted in the final stage. After relaxing the integrality of X_p 's, sub1-1 becomes a large scale linear programming problem as follows:

$$\text{Min. } \sum_p \sum_j \sum_t \sum_k u_{tk} X_{pjtk} - \sum_p W_p X_p$$

subject to

$$\sum_p \sum_j \sum_t t_{pjtk} X_{pjtk} \leq A_k \quad \forall k \quad (3-3)$$

$$\sum_t \sum_k X_{pjtk} - X_p = 0 \quad \forall p, j \quad (3-2)$$

$$0 \leq X_p \leq 1 \quad \forall p \quad (3-17)$$

This large scale linear program has a block diagonal structure. Each block represents a particular part order p . Decomposition principle is used to solve the problem. The problem is decomposed into two separate problems. The one over constraint (3-2) is called the subproblem consisting of P independent sub-subproblems with generalized upper bound (GUB) like structure (Fisher, 1981), which can be easily solved using inspection methods. The one over constraint (3-3) is called the master problem, in which the number of constraints (rows) $(=P+K)$ are much less than the number of variables (columns) $(=PJTK+P)$, assuming that $J_p = J, \forall p$. Therefore, the revised simplex method, incorporating a column generation scheme, is applied to solve the master problem. The details are shown in the Appendix 1.

c. Lagrangian Heuristic

It is well known that the solutions obtained from the relaxed problem are rarely feasible in the original problem. But it often happens

that the Lagrangian solution will be nearly feasible and can be made feasible by a heuristic, which is often called Lagrangian Heuristic (Fisher, 1981, 1985). In our case, LM-1 is solved as two independent problems. One handles part selection and job allocation problems, the other takes care of tool selection and allocation problems. This may give rise to two further problems:

(a) some operations of a selected part have been assigned to a machining center but may not have all the required tools;

(b) a tool has been selected and allocated to a machining center but may not be used by any job.

To investigate these problems, the following manipulations should be performed:

(1) Delete all unnecessary tool assignments to free up tool slots -- this corresponds to (b).

(2) Try to allocate required tools to the jobs which need it -- this corresponds to (a). This may be performed in two ways, i.e., simply assign the required tool to the job if the remaining slots are sufficient on the associated machining center; or, if the remaining slots are not enough, allot some slot resources to the concerned tool from some other tool which has already been allocated to the machining center but has a lower contribution in comparison with the concerned tool.

(3) Adjust job assignments --this is also associated with (a). If the assigned job does not have the required tools, try to allocate it to other tool-machine combinations. If this reallocation is not possible, delete the associated parts.

(4) Re-insert some unselected parts. After conducting 1)--3) and updating the remaining slots and available machining time, it may be possible that the remaining resources (slots and time) are enough to 'absorb' some unselected parts.

The heuristic developed based on the above ideas is described as follows.

Heuristic feas-1

Step 1. Free up slots.

Delete all unnecessary tool assignments to free up slots.

Step 2. Check feasibility

For a particular tool-machine combination (t',k') , check the feasibility of the link constraint (3-4). If feasible, check next (t,k) ; otherwise, continue. If all link constraints are feasible, go to step 5.

Step 3. Adjust Y_{tk}

(If constraint (3-4) is infeasible, $Y_{t,k}$ must be zero).

If $\Delta TQ_k \geq Q_{t,k}$, then

let $Y_{t,k} = 1$ and update (ΔTQ_k) , go to step 2;

else if $CON_{t,k}^* (= \min_{t \in \tau_k} \{ CON_{tk} \}) \leq CON_{t,k}$, then

let $Y_{t,k}^* = 0$, $X_{pjt,k}^* = 0$ ($\forall p, j$), $X_p = 0$ (if $X_{pjt,k} = 0$, for

all j, t, k), $Y_{t,k} = 1$, and update ΔA_k and ΔTQ_k , go to beginning of step 3.

else, if $CON_{t',k'} > CON_{t,k}$, continue.

where, $\tau_k = \{ t \mid Y_{tk} = 1 \}$, CON_{tk} is the contribution (in terms of total W_p handled) of tool t on machining center k ,

$$CON_{tk} = \sum_{p \in \phi_{tk}} W_p, \quad \phi_{tk} = \{ p \mid \max_{1 \leq j \leq J_p} \{ X_{pjtk} \} > 0 \}$$

Step 4. Reallocate jobs

Reallocate the jobs, i.e., (p, j) combinations, having been assigned to (t', k') to other tool-machine combinations. If reallocation succeeds, then update ΔA_k and ΔTQ_k , go to step 2; else, delete all associated parts. Go to next step.

Step 5. Re-insert some unselected parts to the system if possible

(Now, the system may have unused resources (processing time and slots)).

Re-insert part order to the system using the most efficient (in terms of resource consumption) tool-machine combinations sequentially in descending order of W_p values (of unselected parts) until the remaining processing times or slots have been exhausted.

d. Solution Algorithm

Having obtained all the elements, the Lagrangian based method can be summarized below:

Algorithm A-1

- Step 1. Set $n=0$, all Lagrangian multipliers $u_{tk}^0 = 0.01$, upper bound $ZD=MB$ (a big positive number), lower bound $Z^* = 0$.
- Step 2. If $n \geq NITER$ (the maximum number of allowed iterations), stop; otherwise, continue.
- Step 3. Solve sub1-1(relaxed) and sub1-2. The solutions are X_{pjtk}^n , X_p^n and Y_{tk}^n .
- Step 4. Update Lagrangian multipliers using the subgradient method (Fisher, 1981)

$$u_{tk}^{n+1} = \max. \{0, u_{tk}^n - \lambda^n (M Y_{tk}^n - \sum_p \sum_j X_{pjtk}^n)\}$$

$$\lambda^n = \lambda^n (ZD_1^n - Z^*) / \sum_t \sum_k (M Y_{tk}^n - \sum_p \sum_j X_{pjtk}^n)^2$$

the sequence λ^n is defined by setting $\lambda^0 = 2$ and halving λ^n whenever ZD_1^n has failed to decrease in a fixed number of iterations.

Then compute Lagrangian objective

$$ZD_1^n = \sum_p W_p X_p^n - \sum_p \sum_j \sum_t \sum_k u_{tk}^n X_{pjtk}^n + \sum_t \sum_k u_{tk}^n M Y_{tk}^n$$

If $ZD_1^n < ZD$, update ZD and go to the next step; otherwise, go to step 2.

Step 5. If $0 < X_p^n < 1$, let X_p^n and associated X_{pjtk}^n equal 0; else, continue.

Step 6. Check if Y_{tk}^n , X_{pjtk}^n and X_p^n are feasible to problem M-1. If yes, update the best feasible solution and go to step

8; otherwise, continue.

Step 7. Make the solution feasible using heuristic. The feasible objective is Z_1^* . If $Z_1^* \geq Z^*$, update Z^* and go to next step; otherwise, go to step 2.

Step 8. If $(ZD - Z^*)/ZD \leq \epsilon$ (acceptable percentage error), then stop; otherwise, go to step 2.

It should be pointed out that if step 5 in algorithm A-1 is deleted, this algorithm can also be used to handle the situation where X_p is defined as $0 \leq X_p \leq 1$ rather than as a 0-1 integer variable.

3.4.2 Solution Procedure for Model M1-1

The structure of Model M1-1 is similar to that of Model M-1, therefore the solution method in the last section can be adopted with some minor modifications.

a. Lagrangian Relaxation

By dualizing link constraint (3-12), the following Lagrangian problem is obtained (for convenience, maximization form is used).

$$\text{LM1-1: } \text{Max } \left\{ - \sum_{i \in I_1} \sum_{j=1}^J \sum_{t=1}^T \sum_{k=1}^K (u_{tk} + C_{ijtk}) \bar{X}_{ijtk} \right\} + \sum_{t=1}^T \sum_{k=1}^K M u_{tk} Y_{tk}$$

subject to (3-10), (3-11), (3-13), (3-5) and (3-7).

LM1-1 is equivalent to two subproblems below

Subproblem sub1-3

$$\text{Min } \sum_{i \in I_1} \sum_{j=1}^{J_1} \sum_{t=1}^T \sum_{k=1}^K (u_{tk} + C_{1,jtk}) \bar{X}_{1,jtk}$$

Subject to (3-10), (3-11) and (3-13).

Subproblem sub1-4

$$\text{Max } \sum_{t=1}^T \sum_{k=1}^K M u_{tk} Y_{tk}$$

Subject to (3-5) and (3-7).

b. Solution Method of sub1-3 and sub1-4

Sub1-4 is the same as sub1-2 which can be solved using the same method as mentioned earlier.

Sub1-3 is a large scale linear programming problem, which is similar to but simpler than sub1-1. Since X_p type variable disappears in the objective and is replaced by constant 1 in the constraint, and further, (3-17) type constraint is absent, the solution procedure of sub1-3 is simplified. By applying a procedure similar to the one used for sub1-1, sub1-3 can be easily solved. The details are shown in Appendix 2.

c. Lagrangian Heuristic

The heuristic is similar to feas-1 and can be described as follows.

Heuristic feas1-1

Step 1 and 2. Follow step 1-2 of feas-1; here, the link constraint is represented by (3-12).

Step 3. Adjust Y_{tk}

(If link constraint is infeasible, $Y_{t',k}$ must be zero).

If $\Delta TQ_k \geq Q_{t'}$, then

let $Y_{t',k} = 1$ and update (ΔTQ_k) , go to step 2;

else, go to step 4.

Step 4. Reallocate jobs

Reallocate the jobs, i.e., (p,j) combinations, having been assigned to (t', k') to other tool-machine combinations. If reallocation succeeds, then update ΔA_k and ΔTQ_k , go to step 2; else, stop.

d. Solution Algorithm

Algorithm A1-1

Step 1 and 2 Follow step 1--2 of algorithm A-1

Step 3. Solve sub1-3 and sub1-4. The solutions are \bar{X}_{ijtk}^n and Y_{tk}^n .

Step 4. Update Lagrangian multipliers follow the procedure as mentioned in step 4 of algorithm A-1 and compute Lagrangian objective

$$ZD_1^n = \left\{ - \sum_i \sum_j \sum_t \sum_k (u_{tk}^n + C_{ijtk}) \bar{X}_{ijtk}^n \right\} + \sum_t \sum_k u_{tk}^n M Y_{tk}^n$$

If $ZD_1^n < ZD$, update ZD and continue; otherwise go to step 2.

Step 5. Check if Y_{tk}^n and \bar{X}_{1jtk}^n are feasible to problem M1-1.

If yes, update the best feasible solution and go to step 7; otherwise, continue.

Step 6. Try to make the solution feasible using heuristic feasi-1.

If the Lagrangian problem solution is made feasible, the associated objective is Z_1^* . If $Z_1^* \geq Z^*$, update Z^* and go to next step; otherwise, go to step 2.

If the solution of the Lagrangian problem can not be made feasible using feasi-1, go to step 2.

Step 7. If $(ZD - Z^*)/ZD \leq \epsilon$ (acceptable percentage error), then stop; otherwise, go to step 2.

3.4.3 Solution Procedure for Model M1-2

a. Lagrangian Relaxation

The relaxation procedure is the same as in Models M-1 and M1-1. By dualizing link constraint (3-12), the following is obtained

$$\text{LM1-2: } \text{Max } -\left\{ \sum_{i \in I_1} \sum_{j=1}^J \sum_{t=1}^T \sum_{k=1}^K u_{tk} \bar{X}_{1jtk} + H \right\} + \sum_{t=1}^T \sum_{k=1}^K M u_{tk} Y_{tk}$$

subject to (3-10), (3-15), (3-13), (3-5) and (3-7).

LM1-2 consists of the following two independent subproblems.

Subproblem sub1-5

$$\text{Min } \sum_{i \in I_1} \sum_{j=1}^{J_1} \sum_{t=1}^T \sum_{k=1}^K u_{tk} \bar{X}_{ijtk} + H$$

Subject to (3-10), (3-15), (3-13) and (3-16).

Subproblem sub1-6

$$\text{Max } \sum_{t=1}^T \sum_{k=1}^K M u_{tk} Y_{tk}$$

Subject to (3-5) and (3-7).

b. Solution Method of sub1-5 and sub1-6

Sub1-6 is the same as sub1-2 and sub1-4, which can be solved using the same method as mentioned earlier.

Sub1-5 is a large scale linear programming problem. Due to the variable H in the objective function and constraints, the solution procedures here are slightly more complicated than that of sub1-3. Due to the similarity in structure, however, it can also be solved by applying the decomposition principle incorporating the revised simplex method. The procedure is similar to the one used for sub1-1. The details are shown in Appendix 3.

c. Lagrangian heuristic

The heuristic is similar to feas-1 and feas1-1 and is presented as follows.

Heuristic feas1-2

Step 1 and 3. Follow step 1-3 of feas1-1.

Step 4. Reallocate jobs

Try to reallocate the jobs, i.e., (p,j) combinations assigned to (t', k'), to other tool-machine combinations under the restriction of constraint (3-15). If reallocation succeeds, then update $\Delta H_k (= H r_k - \sum \bar{t}_{ijtk} \bar{X}_{ijtk})$ and ΔTQ_k , go to step 2; else, increase H value by an appropriate step length under the restriction of constraint (3-16), and repeat the above procedure. If any job reallocation fails, stop.

Due to the same reason as mentioned in feas1-1, this heuristic does not guarantee that the solution can be made feasible.

d. Solution Algorithm

Algorithm A1-2

Step 1 and 2 Follow step 1--2 of algorithm A-1

Step 3. Solve sub1-5 and sub1-6. The solutions are \bar{X}_{ijtk}^n , H^n and Y_{tk}^n .

Step 4. Update Lagrangian multipliers following the procedure as mentioned in step 4 of algorithm A-1 (section 3.4.1) and compute the Lagrangian objective

$$ZD_1^n = \left\{ - \sum_i \sum_j \sum_t \sum_k u_{tk}^n \bar{X}_{ijtk}^n + H \right\} + \sum_{i,k} \sum_t u_{tk}^n M Y_{tk}^n$$

If $ZD_1^n < ZD$, update ZD and continue, otherwise go to step 2.

Step 5 - 7 Follow step 5-7 of algorithm A1-1.

Because all selected parts have to be processed, no job can be deleted in $feas1-1$ and $feas1-2$. Therefore, it is not guaranteed that the solution can be made feasible by using the heuristic $feas1-1$ (or $feas1-2$) alone. In this case, the iteration continues in algorithm A1-1 (or A1-2) until a feasible solution is obtained or the maximum number of iterations is reached. If the maximum number of iterations has been reached but still no feasible solution is obtained, the maximum number of iterations may be increased. If the maximum number of iterations is not allowed to be increased, the iteration is terminated and leads to a conclusion that the secondary objective can not be further improved within the allowed maximum number of iterations for algorithm A1-1 or A1-2.

CHAPTER IV

THE COMBINED PART SELECTION AND MACHINE LOADING PROBLEM IN FMS II

4.1 Problem Description

This chapter addresses the combined PS and ML problem in FMS II where tool transportation devices are not available; batch sizes vary widely or are highly unstable. Most FMSs in use do not have the tool transportation devices (Hwang and Shogan, 1989). Because of this, as well as the unstable batch size resulting from the less predictable order size and lead time requirements, it is obviously important to develop an integrated approach to the PS and ML problems in this type of FMS.

In this case, to reduce the number of times that a machine is stopped to change tools, spare tools of same tool type should be loaded into the magazine (Gray et al. 1988) before production. So the tooling decision should consider not only the selection of specific types of tools for a machining center but also the allocation of required number of spare tools of each tool type to a machining center. Keeping this in mind, a set of associated models are developed in the next section based on following assumptions:

- 1) The compatibility of each tool-machine and tool-job combination is given.
- 2) The data related to processing cost and time of each job-tool-machine combination are available.

- 3) Enough spare tools of each required tool type are available.
- 4) The operation sequence of each type of part is given. The alternative operation sequences are not considered.
- 5) Order sizes are known and are fixed at the beginning of the period. The tool magazines are unchangeable and are fixed to the machines (Sarin and Chen 1987).

The notations used in this chapter are listed as follows:

Subscripts

- $i = 1, \dots, I$ part type
 $j = 1, \dots, J_p$ operation
 $k, = 1, \dots, K$ machining center
 $p = 1, \dots, P$ part order
 $t = 1, \dots, T$ tool type

Variables

- H = makespan (throughput time)
 V_{tk} = number of tools of tool type t assigned to machining center k
 \bar{X}_{ijtk} = proportion of operation j of part type i to be processed using tool t on machining center k .
 $X_p = \begin{cases} 1, & \text{if part order } p \text{ is selected for FMS} \\ 0, & \text{otherwise} \end{cases}$
 X_{pjtk} = proportion of operation j of part order p to be processed using tool t on machining center k

Parameters

A	= planning horizon
A_k	= available machining time on machining center k , $A_k = A r_k$
ΔA_k	= remaining available processing time at machining center k
C_{ijtk}	= machining cost to process operation j of part order i using tool t on machining center k
L_t	= tool life of tool type t
Q_t	= number of slots needed by tool t
r_k	= maximum utilization limit of machining center k
\bar{t}_{ijtk}	= machining time needed to process operation j of part type i using tool t on machining center k
t_{pjtk}	= machining time needed to process operation j of part order p using tool t on machining center k
TQ_k	= number of slots available at machining center k
ΔTQ_k	= remaining slot space at machining center k
W_p	= weight of part order p

Lagrangian relaxation and computation related parameters

ϵ	= the maximum acceptable percentage error
λ^n	= a scalar in iteration n , $0 < \lambda_n \leq 2$
S_u^n	= a positive scalar step size in iteration n
u_{tk}^n	= Lagrangian multiplier in iteration n
Z^*	= the best (highest) lower bound (on the objective value of the Lagrangian problem) obtained so far
Z_1	= the lower bound (on the objective value of the Lagrangian

problem) obtained in the current iteration

ZD = the best (lowest) upper bound (on the objective value of the Lagrangian problem) obtained so far

ZD_1^n = the upper bound (on the objective value of the Lagrangian problem) obtained in the current iteration

Sets

I_1 = selected part type set, each of the part types has at least one part order having been selected

S_1 = a subset of (t,k) combinations for which the tooling requirement is less than the available tooling assigned

S_2 = a subset of (t,k) combinations for which the tooling requirement equals the available tooling assigned

S_3 = a subset of (t,k) combinations for which the tooling requirement is greater than the available tooling assigned

4.2 Model Formulation

The models are developed to answer following decision problems:

- 1) part selection
- 2) job allocation
- 3) tool type selection and assignment
- 4) process route selection
- 5) determination of number of spare tools for each machine-tool (type) combination.

The same primary and secondary objectives as used in last chapter are adopted.

The models are presented as follows.

Model for Primary Objective

Model (M-2)

$$\text{Max } \sum_{p=1}^P W_p X_p \quad (4-1)$$

Subject to

$$X_p - \sum_{t=1}^T \sum_{k=1}^K X_{p,jtk} = 0 \quad \forall p$$

$$j=1, 2, \dots, J_p \quad (4-2)$$

$$\sum_{p=1}^P \sum_{j=1}^{J_p} \sum_{t=1}^T t_{p,jtk} X_{p,jtk} \leq A_k \quad \forall k \quad (4-3)$$

$$\sum_{p=1}^P \sum_{j=1}^{J_p} t_{p,jtk} X_{p,jtk} - L_t V_{tk} \leq 0 \quad \forall t, k \quad (4-4)$$

$$\sum_{t=1}^T Q_t V_{tk} \leq TQ_k \quad \forall k \quad (4-5)$$

$$X_p = 0, \text{ or } 1 \quad \forall p \quad (4-6)$$

$$V_{tk} \geq 0, \text{ integer variable} \quad \forall t, k \quad (4-7)$$

$$0 \leq X_{p,jtk} \leq 1 \quad \forall p, t, k \quad (4-8)$$

$$j=1, \dots, J_p$$

Constraint (4-2) states that the total amount of part order p processed at all feasible machining centers using all feasible tools should be equal for all operations. The available processing time and

magazine capacity restrictions are represented by constraints (4-3) and (4-5), respectively. Constraint (4-4) ensures that all job and tool assignments should be feasible. Constraint (4-6) says that a part order is either entirely selected for immediate processing or totally rejected for immediate processing. Constraint (4-7) represents the integrality of variable V_{tk} . The bounds of X_{pjtk} are imposed by constraint (4-8).

Based on the same reasoning as stated in Chapter 3, the tool magazine capacity is considered as a constraint in model M-2.

Similar to the discussion in Chapter 3, X_p in model M-2 is a 0-1 integer variable since all parts within a part order generally have the same due date requirement. But if the parts within a part order are allowed to be processed in different periods, model M-2 can also be applied. The only modification is to replace constraint (4-6) with $0 \leq X_p \leq 1$.

Model for Secondary Objective 1 (Minimize Machining Cost)

Model (M2-1)

$$\text{Min } \sum_{i \in I_1} \sum_{j=1}^{J_1} \sum_{t=1}^T \sum_{k=1}^K C_{ijtk} \bar{X}_{ijtk} \quad (4-9)$$

Subject to

$$\sum_{t=1}^T \sum_{k=1}^K \bar{X}_{i,j,t,k} = 1 \quad i \in I_1, \quad j=1, \dots, J_1 \quad (4-10)$$

$$\sum_{i \in I_1} \sum_{j=1}^{J_1} \sum_{t=1}^T \bar{t}_{i,j,t,k} \bar{X}_{i,j,t,k} \leq A_k \quad \forall k \quad (4-11)$$

$$\sum_{i \in I_1} \sum_{j=1}^{J_1} \bar{t}_{i,j,t,k} \bar{X}_{i,j,t,k} - L_t V_{t,k} \leq 0 \quad \forall t, k \quad (4-12)$$

$$0 \leq \bar{X}_{i,j,t,k} \leq 1 \quad i \in I_1, j=1, \dots, J_1, \quad \forall t, k \quad (4-13)$$

and (4-5), (4-7).

where, I_1 is a subset of part types, among which each part type has at least one selected part order. Constraints (4-10)--(4-12) embody the restrictions represented by constraints (4-2)--(4-4) for the selected part types. Constraint (4-13) maps constraint (4-8) to the selected subset of part types.

Model for Secondary Objective 2 (Minimize Makespan)

Model (M2-2)

$$\text{Min } H \quad (4-14)$$

Subject to (4-5), (4-7), (4-10), (4-12), (4-13) and

$$\sum_{i \in I_1} \sum_{j=1}^J \sum_{t=1}^T \bar{t}_{ijtk} \bar{X}_{ijtk} \leq H r_k \quad \forall k \quad (4-15)$$

$$H \leq A \quad (4-16)$$

Constraint (4-15) ensures that the available processing time must not be exceeded at each machining center. Constraint (4-16) states that makespan should be less than or equal to the predetermined production horizon.

As mentioned in Chapter 3, if the primary objective is to maximize throughput, the secondary objective 2 (minimize makespan) is applied only when a) both magazine capacity and available machining time constraints are loose, or b) the magazine capacity constraint is tight but the available machining time constraint is loose.

Also, as mentioned in Chapter 3, this set of models is a planning tool which focuses on part selection and machine loading problems. The job sequencing issue is not considered. Other reasons have been stated in Chapter 3. Again, it should be mentioned that the minimum makespan obtained from the secondary objective model may be affected by different scheduling methods.

It may be worthwhile to point out the differences and similarities between models M-1 and M-2. Mathematically, the differences between the two models are:

1) decision variables:

Y_{tk} (0-1 integer variable) in M-1

V_{tk} (general integer variable) in M-2

2) constraints:

$$\sum \sum X_{pjtk} - M Y_{tk} \leq 0 \quad (3-4) \quad (\text{in } M-1)$$

$$\sum \sum t_{pjtk} X_{pjtk} - L_t V_{tk} \leq 0 \quad (4-4) \quad (\text{in } M-2)$$

By replacing L_t with M , and V_{tk} with Y_{tk} , in (4-4), equation (4-4) will have a form similar to (3-4). Therefore, mathematically, M-2 may be considered as a general form of M-1 if the meaning of M is redefined. However, model M-1 explicitly represents the problem in FMS I in the physical sense and is also computationally easier to handle. This is because when M-2 replaces M-1, the vector C_p (see Appendix 1) will have to be recalculated by replacing u_{tk} with $u_{tk} * t_{pjtk}$. For example, if $P=50$, $J=5$, $T=50$, and $K=5$, it will need 62,500 more multiplications. Hence, model M-1 is a preferred form of the problem in FMS I.

4.3 Illustrative Example

To illustrate the attributes of the models developed, an example, along the same lines as those used by the researchers mentioned in Chapter 3 is presented as follows.

Example 4.1

Consider an FMS with three machining centers. The planning horizon is 125 hours. The maximum utilization limit of each machining center is

80%. The available processing time at each machining center, therefore, is 100 hours. The magazine size at each machining center is 80 tool slots. 15 types of tools are available. There is no tool transportation device. Six candidate part types (each has only one part order), each with 3 operations, are considered for selection. The order sizes of the six types of parts are: 20, 30, 50, 10, 30 and 10, respectively. The tool-machine, tool-job compatibility, slot requirement of each tool, and machining cost and time data are the same as those in the example of Chapter 3. The expected tool life of each tool type is 3 hours.

The primary objective is to maximize throughput. The secondary objective is to reduce machining cost, or makespan, as much as possible.

Initially the primary objective problem has 50 general integer variables, 810 continuous variables and 69 constraints. After deleting infeasible tool-machine and tool-job combinations, the problem size reduces to 26 general integer variables, 50 continuous variables and 44 constraints. The problem sizes of secondary objectives depend on the solution of the primary objective, i.e., number of selected part types. Generally the size of the secondary problem is smaller than that of the primary objective. The example problem has been solved using the LINDO micro-computer package. The unnecessary tool assignments are eliminated by checking $\sum_p \sum_j X_{pjtk}$ values as mentioned in Chapter 3. The solutions of primary and secondary objectives are

listed in Table 4.1.

Table 4.1 shows that both PS and ML decisions, i.e., the decisions about part selection, job allocation, tool type selection and assignment, process route selection (This is automatically obtained from the solution of the model) and number of tools for each machine-tool (type) combination, can be concurrently made based on the solutions. It is also noticed that only solving the primary objective problem can provide a solution for all the above-mentioned decisions. If the decision maker is not fully satisfied with the resultant solution or if capacity has still not been fully utilized, then the secondary objective is introduced. In such a case, the primary objective provides the solution to the PS problem while ML decision is further refined by improving upon the secondary objective.

From Table 4.1, it is once again shown that optimizing the secondary objective can considerably reduce machining cost (14.8%) or makespan (8.7%) while keeping the maximum throughput unchanged.

The job and tool allocation pattern, and number of tools of each machine-tool (type) combination are different before and after optimizing the secondary objective. This once again shows that the improvement of the secondary objective can be achieved by rearranging job and tool allocation pattern. In other words, flexibility can be converted to economic benefit.

The effects of available machining time and magazine capacity on

system performance are observed through additional computations. In Chapter 3, it is observed that magazine capacity (available machining time) has a significant effect on the part selection decision but this effect may be bounded by available machining time(magazine capacity). Similar observation can be made based on Figures 4.1 and 4.2.

Pri. obj.	MC	Job assignment (%)	Tool assignment	Cost (\$)	Planning horizon (makespan) (hrs)
		(1)	(2)		
	M1	J22/74, J31/82, J41/90, J43/75, J51/100, J53/44	T1/3, T2/5, T4/3, T7/11, T11/11		
Max. throughput	M2	J31/13, J32/15, J33/100, J41/10, J42/100, J43/25, J52/100, J53/20	T1/2, T4/1, T6/11, T8/10, T10/3, T12/1, T14/6	48,733	125 (115)
	M3	J21/100, J22/26, J23/100, J31/5, J32/85, J53/36	T3/9, T5/6, T7/6, T9/2, T15/4		

Part types selected: 2, 3, 4, 5. Throughput: 120

Process routes: part 2: M3 { M1 } M3, (3) part 3: M2 { M1 } M2, M3 M3
 part 4: M1 { M2 } M1, M2 M2
 part 5: M1-M2 { M1 } M2, M3 M3

- (1) J22/74 means 74% of job J22.
 (2) T1/3 means 3 tools of tool type 1.
 (3) Indicates operation 1,2 and 3 of part 2 are assigned to machining centers M3, M1 and M3, and M3, respectively.

Table 4.1. Part selection and loading solution for example 4.1.

Table 4.1. continued

Sec. obj.	MC	Job assignment (%)	Tool assignment	Cost (\$)	makespan (hrs)
	M1	J21/100, J31/65, J43/100, J51/100	T1/2, T2/4, T3/10, T4/4, T11/7		
Min. Cost	M2	J31/20, J41/100, J42/100, J52/100, J53/100	T1/6, T6/14, T8/10, T10/3	41,525	121
	M3	J22/100, J23/100, J31/15, J32/100, J33/100	T3/2, T5/5, T7/17, T9/2		
Process routes:	part 2: M1-M2-M3, part 3: $\begin{matrix} M1 \\ M2 \end{matrix} \rightarrow M3-M3$ $M3$ part 4: M2-M2-M1, part 5: M1-M2-M2				
	M1	J21/16, J31/93, J33/60, J41/100, J43/75, J51/100	T1/3, T2/4, T3/2, T4/3, T7/6, T11/13		
Min. makespan	M2	J31/8, J42/100, J43/25, J52/100, J53/98	T1/1, T4/1, T6/13, T8/10, T10/3	44,793	105
	M3	J21/84, J22/100, J23/100, J32/100, J33/40, J53/2	T3/7, T5/11, T7/5, T9/2		
Process routes:	part 2: $\begin{matrix} M1 \\ M3 \end{matrix} \rightarrow M3-M3$, part 3: $\begin{matrix} M1 \\ M2 \end{matrix} \rightarrow M3 \rightarrow M3$ $M2$ part 4: $M1-M2 \rightarrow \begin{matrix} M1 \\ M2 \end{matrix}$, part 5: $M1-M2 \rightarrow \begin{matrix} M2 \\ M3 \end{matrix}$				

(1) T1/3 means 3 tools of tool type 1 are allocated

Table 4.1. Part selection and loading solution
for example 4.1.(continued)

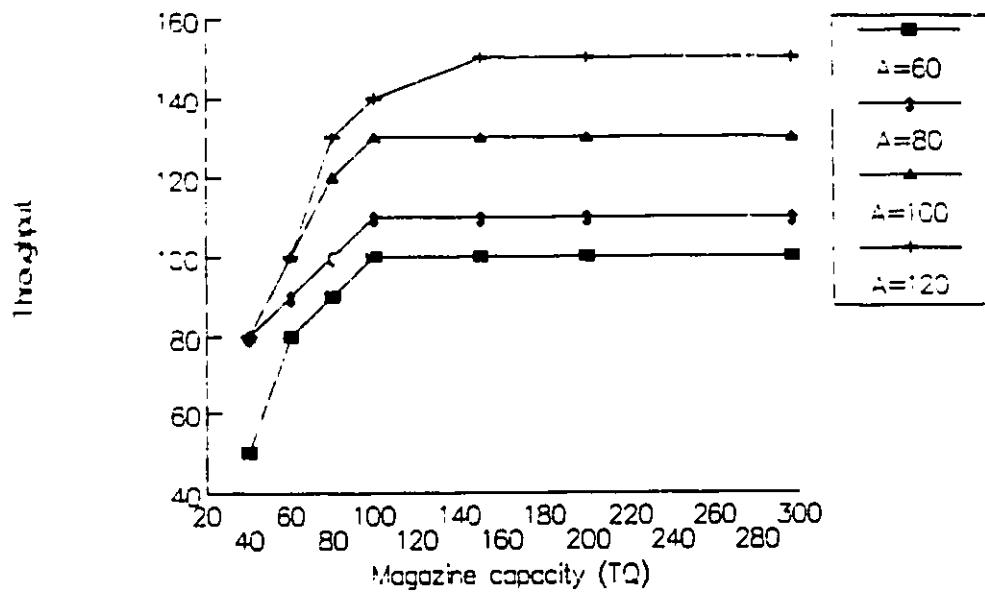


Figure 4.1 Effect of tool magazine capacity on throughput (for example 4.1)

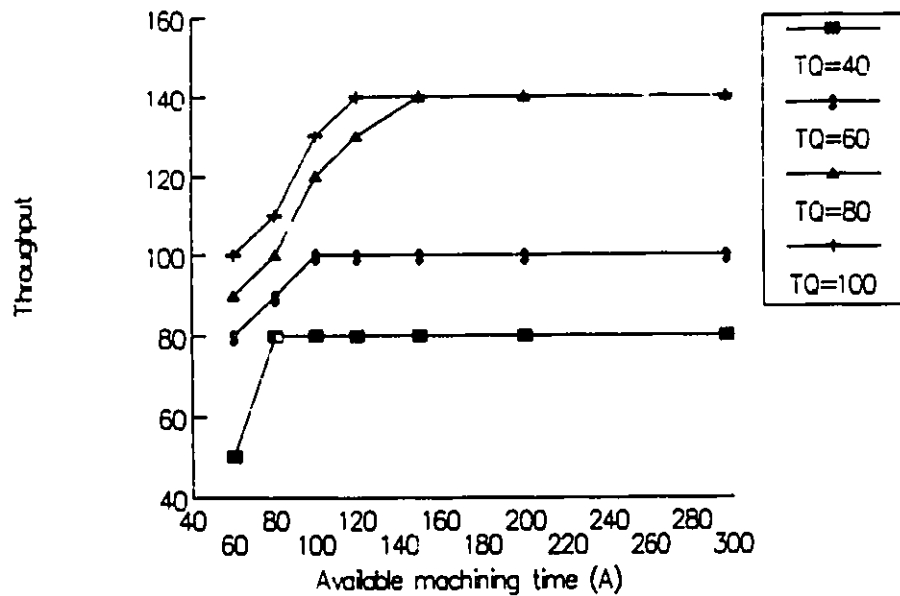


Figure 4.2 Effect of available machining time on throughput (for example 4.1)

4.4 Solution Methodology

In general, the size of model (M-2) is (assuming $J_p = J$, for all p):

Number of general integer variables = $P+TK$

Number of continuous variables = $PJTK$

Number of constraints = $JP+2K+TK$

A problem of this size is very difficult to solve using existing commercial packages. Lagrangian relaxation based solution methods have been proposed to solve relatively large sized problems for models (M-2), (M2-1) and (M2-2).

4.4.1 Solution Procedure for Model M-2

a. Lagrangian Relaxation of Model (M-2)

By relaxing the link constraint (4-4), the following Lagrangian problem is obtained

$$\text{LM-2: } \quad \text{Max} \quad \sum_{p=1}^P W_p X_p - \sum_{p=1}^P \sum_{j=1}^J \sum_{t=1}^T \sum_{k=1}^K u_{tk} t_{pjtk} X_{pjtk} + \sum_{t=1}^T \sum_{k=1}^K u_{tk} L_t V_{tk}$$

Subject to (4-2), (4-3), (4-5) -- (4-8).

This problem is equivalent to the following two independent sub-problems:

Subproblem sub2-1

$$\text{Min} \quad \sum_{p=1}^P \sum_{j=1}^J \sum_{t=1}^T \sum_{k=1}^K u_{tk} t_{pjtk} X_{pjtk} - \sum_{p=1}^P W_p X_p$$

Subject to (4-2), (4-3), (4-6) and (4-8).

Subproblem sub2-2

$$\text{Max} \sum_{t=1}^T \sum_{k=1}^K u_{tk} L_t V_{tk}$$

Subject to (4-5) and (4-7).

b. Solution Methods of sub2-1 and sub2-2

Sub2-1 is essentially same as sub1-1 in Chapter 3.

Subproblem sub2-2 consists of K independent single general integer knapsack problems. All the K single general integer knapsack problems are unbounded and can be solved using the algorithm developed by Martello and Toth (1990).

c. Lagrangian Heuristic

In this case, the Lagrangian problem solution can be partitioned into three sets defined by

$$S_1 = \{t, k \mid \sum_p \sum_j t_{pjtk} X_{pjtk} < L_t V_{tk} \},$$

$$S_2 = \{t, k \mid \sum_p \sum_j t_{pjtk} X_{pjtk} = L_t V_{tk} \},$$

$$S_3 = \{t, k \mid \sum_p \sum_j t_{pjtk} X_{pjtk} > L_t V_{tk}\}.$$

The heuristic is presented below.

Heuristic feas-2

Step 1. Classify the tool-machine combinations

If a tool-machine combination (t, k) belongs to S_1 , then go to step 2;
else, if (t, k) belongs to S_2 , then check the next (t, k) ;
else, if (t, k) belongs to S_3 , then go to step 3.

Step 2. Delete unnecessary tools assigned to each machine

Delete redundant tools for each (t, k) combination, until

$$V_{tk} = \lceil (\sum_p \sum_j t_{pjtk} X_{pjtk}) / L_t \rceil$$

then go to step 1.

(where, $\lceil (\sum_p \sum_j t_{pjtk} X_{pjtk}) / L_t \rceil$ = the smallest integer which is larger than or equal to $(\sum_p \sum_j t_{pjtk} X_{pjtk}) / L_t$)

Step 3. Reallocate jobs

Try to reallocate the job with lowest W_p value to other tool-machine combinations. If reallocation succeeds, then update ΔA_k , go to step 1;
else, reallocate the job with next lowest W_p value and repeat the procedure. If it is still infeasible after performing all associated job allocations, go to next step.

Step 4. Discard some jobs and associated parts

Discard the job with lowest W_p value and associated part. Also delete all jobs of the discarded part that have been allocated to other tool-machine combinations. After this manipulation, some tools may become redundant. If so, delete the redundant tools and then update ΔA_k and ΔTQ_k . Repeat this procedure until the related link constraint is feasible.

Step 5. Re-insert some unselected parts if possible

The system may have some resources remaining from step 4, whereby it may be possible to add some more part orders to the system.

In such a case, re-insert parts into the system using most efficient (in the sense of resource consumption) tool-machine combinations sequentially in descending order of W_p values (of the unselected parts) until the remaining processing times or slots are exhausted.

d. Solution Procedures

A step by step algorithm of the solution procedure is as follows:

Algorithm A-2

Step 1. Set $n=0$, all Lagrangian multipliers $u_{tk}^0 = 0.01$, objective upper bound $ZD=MB$ (a big positive number), lower bound $Z^* = 0$.

Step 2. If $n \geq NITER$ (maximum number of iterations), stop; otherwise, continue.

Step 3. Solve sub2-1 (relaxed) and sub2-2. The solution is X_{pjtk}^n X_p^n and V_{tk}^n .

Step 4. Update Lagrangian multipliers using the following equations

$$u_{tk}^{n+1} = \max. \{0, u_{tk}^n - su (L_t V_{tk}^n - \sum_p \sum_j t_{pjtk} X_{pjtk}^n) \}$$

$$su^n = \lambda^n (ZD_1^n - Z^*) / \sum_{t,k} (L_t V_{tk}^n - \sum_p \sum_j t_{pjtk} X_{pjtk}^n)^2$$

the sequence λ^n is defined by setting $\lambda^0 = 2$ and halving λ^n whenever ZD_1^n has failed to decrease in a fixed number of iterations. Then, compute the Lagrangian objective

$$ZD_1^n = \sum_p W_p X_p^n - \sum_p \sum_j \sum_{t,k} u_{tk}^n t_{pjtk} X_{pjtk}^n + \sum_{t,k} u_{tk}^n L_t V_{tk}^n$$

If $ZD_1^n < ZD$, update ZD and go to next step; otherwise, go to step 2.

Step 5. If $0 < X_p^n < 1$, let X_p^n and associated X_{pjtk}^n equal 0, else, continue.

Step 6. Check if V_{tk}^n , X_{pjtk}^n and X_p^n are feasible to problem M-2. If yes, update the best feasible solution and go to step 8; otherwise, continue.

Step 7. Make the solution feasible using heuristic feas-2. The feasible objective is Z_1^* . If $Z_1^* \geq Z^*$, update Z^* and go to next step; otherwise, go to step 2.

Step 8. If $(ZD - Z^*)/ZD \leq \epsilon$ (acceptable percentage error), then stop; otherwise, go to step 2.

It should be mentioned that if constraint (4-6) is replaced by $0 \leq X_p \leq 1$, the algorithm should be modified by deleting step 5.

4.5.2 Solution Procedure for Model M2-1

a. Lagrangian Relaxation

If constraint (4-12) is relaxed, the Lagrangian problem of model M2-1 can be obtained as follows:

$$\text{LM2-1: Max } \left\{ - \sum_{i \in I_1} \sum_{j=1}^J \sum_{t=1}^T \sum_{k=1}^K (C_{ijtk} + u_{tk} \bar{t}_{ijtk}) \bar{X}_{ijtk} \right\} + \sum_{t=1}^T \sum_{k=1}^K u_{tk} L_t V_{tk}$$

subject to (4-10), (4-11), (4-13), (4-5) and (4-7).

Two equivalent independent subproblems can be obtained from LM2-1 as follows.

Subproblem sub2-3

$$\text{Min. } \sum_{i \in I_1} \sum_{j=1}^J \sum_{t=1}^T \sum_{k=1}^K (C_{ijtk} + u_{tk} \bar{t}_{ijtk}) \bar{X}_{ijtk}$$

Subject to (4-10), (4-11) and (4-13).

Subproblem sub2-4

$$\text{Max. } \sum_{t=1}^T \sum_{k=1}^K u_{tk} L_t V_{tk}$$

Subject to (4-5) and (4-7).

b. Solution Method of sub2-3 and sub2-4

Sub2-3 has same structure as sub1-3 and, therefore, can be solved using the same solution procedure of sub1-3, as is shown in Appendix 2.

Sub2-4 is identical to sub2-2, which can be solved following the solution method for sub2-2.

c. Lagrangian Heuristic

Heuristic feas2-1

This heuristic includes three steps which are the same as step 1 -- 3 of heuristic feas-2 but the link constraint here is (4-12). If step 3 fails, i.e., not all infeasible jobs can be reallocated, then stop.

d. Solution algorithm

Algorithm A2-1

Step 1 and 2. Follow step 1 - 2 of algorithm A-2.

Step 3. Solve sub2-3 and sub2-4 following the methods of solving sub1-3 and sub2-2, respectively. The solutions are \bar{X}_{ijtk}^n and V_{tk}^n .

Step 4. Update Lagrangian multipliers following the procedure in step 5 of algorithm A-2 and compute the Lagrangian objective value

$$ZD_1^n = \left\{ - \sum_i \sum_j \sum_t \sum_k (C_{ijtk} + u_{tk}^n \bar{t}_{ijtk}) \bar{X}_{ijtk}^n + \sum_t \sum_k u_{tk}^n L_t V_{tk}^n \right.$$

If $ZD_1^n < ZD$, update ZD and go to next step; otherwise, go to step 2.

Step 5. Check if V_{tk}^n and \bar{X}_{ijtk}^n are feasible to problem M2-1. If yes, update the best feasible solution and go to step 7; otherwise, continue.

Step 6. Try to make the solution feasible using heuristic feas2-1. If the solution is made feasible, the corresponding objective value is Z_1^* . If $Z_1^* \geq Z^*$, update Z^* and go to next step; otherwise, go to step 2. If the Lagrangian solution can not be made feasible using feas2-1, go to step 2.

Step 7. If $(ZD - Z^*)/ZD \leq \epsilon$ (acceptable percentage error), then stop; otherwise, go to step 2.

4.5.3 Solution Procedure for Model M2-2

a. Lagrangian Relaxation

After relaxing constraint (4-12), it gives

$$LM2-2: \quad \text{Max } -\left\{ \sum_{i \in I_1} \sum_{j=1}^J \sum_{t=1}^T \sum_{k=1}^K u_{tk} \bar{t}_{ijtk} \bar{X}_{ijtk} + H \right\} + \sum_{t=1}^T \sum_{k=1}^K u_{tk} L_t V_{tk}$$

subject to (4-10), (4-13), (4-15), (4-16), (4-5) and (4-7).

This problem is equivalent to the following two subproblems:

Subproblem sub2-5

$$\text{Min } \sum_{i \in I_1} \sum_{j=1}^J \sum_{t=1}^T \sum_{k=1}^K u_{tk} \bar{t}_{ijtk} \bar{X}_{ijtk} + H$$

subject to (4-10), (4-13), (4-15) and (4-16).

Subproblem sub2-6

$$\text{Max } \sum_{t=1}^T \sum_{k=1}^K u_{tk} L_t V_{tk}$$

subject to (4-5) and (4-7).

b. Solution Methods of sub2-5 and sub2-6

Sub2-5 has a structure similar to sub1-5 and, therefore, can be solved following the same procedure as presented in Appendix 3.

The solution method of sub2-6 is the same as that of sub2-2 in that they are identical.

c. Lagrangian Heuristic

Heuristic feas2-2

Step 1 and 3. Follow step 1--3 of heuristic feas2-1.

Step 4. Follow step 4 of heuristic feas 1-2.

d. Solution Algorithm

Algorithm A2-2

Step 1 and 2. Follow step 1--2 of algorithm A-2.

Step 3. Solve sub2-5 and sub2-6.

Step 4. Update Lagrangian multipliers following the procedure in step 5 of algorithm A-2 and compute ZD_1^n . If $ZD_1^n < ZD$, update ZD and continue, otherwise go to step 2.

Step 5 and 7. Follow step 5 - 7 of algorithm A2-1.

The feasible solution may not be obtained by using heuristic feas2-1 (or feas2-2) alone. If this is the case, the iterations of algorithm A2-1 (or A2-2) continues until a feasible solution is obtained or the maximum number of iterations is reached. If the maximum number of iterations is reached without obtaining a feasible solution, the allowed maximum number of iterations may be increased. If the feasible solution is still not obtained within the increased maximum number of iterations or if the maximum number of iterations is not allowed to be increased, the secondary objective will not be improved.

CHAPTER V

THE COMBINED PART SELECTION AND MACHINE LOADING PROBLEM IN THE HYBRID SYSTEM (HS)

This chapter addresses the combined PS and ML problem in a hybrid manufacturing system, namely, a system where flexible manufacturing sub-system (or flexible manufacturing cell--FMC) and conventional manufacturing sub-system (CMS) (e.g., job shop, or a batch production sub-system with process layout facilities) exist simultaneously. The FMS portion of the system can be either a FMS I type or FMS II type sub-system. The problems involved in this type of system are described in the next section.

5.1 Problem Description

In a hybrid system, the combined PS and ML problem can be classified into two cases based on the resources availability in the CMS, i.e.:

Case 1. The production tasks can not be finished using FMS alone within the immediate following period but the remaining can be finished by CMS.

Case 2. The production requirements can not be finished by both FMS and CMS within the immediate following period.

In either case some or all of the following three aspects should be addressed:

1. Part selection
2. Machine loading (job and tool allocation), and
3. Load sharing between CMS and FMS sub-systems.

In case 1, there is no need to select part types or part orders since all parts will be finished within next period anyway. In stead, the problem becomes one of allocating the part types between the two sub-systems and loading the jobs in each of the sub-systems, i.e., only two of the three aspects: machine loading and load sharing are to be considered. In case 2, since not all parts can be finished even when both sub-systems are used, due to the production capacity limitation, parts should, therefore, be selected for the whole HS based on the importance (expressed as weight) of each part order. This indicates that all three aspects mentioned above have to be considered in case 2.

The related work has been partially investigated by some researchers (Hutchinson and Holland, 1982; Gaimon, 1985; Avonts *et al.* 1988; Gupta *et al.* 1988; and Kayaligil, 1990). However, since their work focus on load sharing aspect, some limitations exist. E.g., in the work by Avonts *et al.*(1988) and Kayaligil (1990), the following problems exist:

(a) Focus on only load sharing, consider only a given set of part types or fixed part-mix. No part selection or machine loading are considered.

(b) Assume all jobs can be finished within the conventional and flexible systems.

(c) Since machine loading problem is not considered, the alternative ways (alternative machines, alternative tools, routes) for manufacturing each part are ignored. This means that the inherent flexibility of the manufacturing system is not effectively utilized.

From (a) --(c) it is clear that the previous work only partially investigated the problem, i.e., only aspect 3 of case 1 is addressed. Further work involved in aspect 1 and 2 of case 1 and all three aspects of case 2 are needed. In this research all of the three aspects in both cases will be addressed and a integrated approach will be applied to obtain a set of consistent solution.

Further, to make better use of the inherent flexibility of an FMS, the commonly used assumption that a operation can be assigned to only one machine or machining center is dropped in this research.

Before formulating the problems, the following assumptions are made:

(1) No inter-system move is allowed. In other word, once a part is assigned to a sub-system, FMS or CMS, all its operations will be performed within the designated sub-system.

(2) (For case 2) a part order is either entirely selected or totally rejected for processing.

Other assumptions related to the FMS portion of the system are identical with those mentioned in previous two chapters.

5.2 Model Formulations

The models are developed to make the following decisions.

- 1) Load sharing between the two sub-systems.
- 2) Part selection -- for case 2.
- 3) Job allocation.
- 4) Tool selection and assignment -- for FMS sub-system.
- 5) Process route selection.
- 6) Determination of number of spare tools of each tool type on each machining center if the FMS sub-system is similar to FMS II.

The following notations are used in this chapter:

Subscripts

- $i = 1, \dots, I$ part type
- $j = 1, \dots, J_p$ operation
- $k = 1, \dots, K$ machining center (in FMS), or work center (in CMS)
- $p = 1, \dots, P$ part order
- $t = 1, \dots, T$ tool type

Variables

V_{tk} = number of tools of tool type t assigned to machining center k in FMS sub-system

$$\bar{X}_i = \begin{cases} 1, & \text{if part type } i \text{ is selected for FMS sub-system} \\ & \text{in a hybrid system} \\ 0, & \text{otherwise} \end{cases}$$

$$\bar{X}'_i = \begin{cases} 1, & \text{if part type } i \text{ is selected for CMS sub-system} \\ & \text{in a hybrid system} \\ 0, & \text{otherwise} \end{cases}$$

\bar{X}_{ijtk} = proportion of operation j of part type i to be processed using tool t on machining center k in FMS sub-system.

$$X_p = \begin{cases} 1, & \text{if part order p is selected for FMS} \\ 0, & \text{otherwise} \end{cases}$$

$$X'_p = \begin{cases} 1, & \text{if part order p is selected for CMS sub-system} \\ & \text{in a hybrid system} \\ 0, & \text{otherwise} \end{cases}$$

X'_{pjk} = proportion of operation j of part order p assigned to work center k in CMS sub-system.

X_{pjtk} = proportion of operation j of part order p to be processed using tool t on machining center k in FMS sub-system.

$$Y_{tk} = \begin{cases} 1, & \text{if tool t is assigned to machining center k} \\ 0, & \text{otherwise} \end{cases}$$

Parameters

A_k = available machining time on machining center k

HC_i = work in process inventory cost (per unit time) of all parts of type i

$K(i, j)$ = number of feasible work centers for job (i, j) in CMS

L_t = tool life of tool type t

ΔL = the lead time difference between FMS and CMS sub-system

M = a positive integer, indicating an upper bound on the number of operations that can be assigned to a tool-machine combination

- Q_t = number of slots needed by tool t
 R_{ijk} = the cost per major setup for operation j of part i at work center k in CMS sub-system
 SC_i = average setup cost of operation j of part type i in CMS,

$$SC_i = \sum_{j=1}^{J_i} \sum_{k \in K_{ij}} R_{ijk} / K(i, j)$$

 \bar{t}_{ijtk} = machining time needed to process operation j of part type i using tool t on machining center k in FMS
 t'_{pjk} = machining time needed to process operation j of part order p at work center k in CMS
 $t_{p,jtk}$ = machining time needed to process operation j of part order p using tool t on machining center k in FMS
 TQ_k = number of slots available at machining center k
 W_p = weight of part order p

Sets

- K_c = a set of machines available in the CMS sub-system of a hybrid system
 K_f = a set of machining centers in the FMS sub-system of a hybrid system.
 K_{ij} = a set of machines (work centers) in CMS sub-system of a hybrid system which can perform job (i, j) .

a. Model for Case 1

Compared with CMS, two of the most obvious advantages of FMS are low setup cost (almost zero) and short lead time which in turn means low work in process inventory cost. By assigning the parts with high setup

requirements (or high setup cost) and/or high holding cost to FMS, the cost can be considerably reduced. Therefore, the primary objective is to maximize the cost savings resulting from proper allocation of the parts between FMS and CMS.

The secondary objectives of the FMS sub-system are the same as before, i.e., to minimize processing cost or minimize makespan.

For the CMS sub-system the secondary objective is to minimize total production cost, including machining cost, material handling cost, setup cost, machine idle cost and the penalty cost toward long makespan. For the CMS portion the makespan consideration is incorporated in the total production cost for two reasons:

- (a) the part move distance from one work center to another work center is generally long and the setup cost is high. Minimizing makespan may cause more operation splittings and consequently higher material handling and setup costs;

- (b) the main portion of cost in CPS is the variable cost rather than fixed cost since the machinery generally is not as expensive as in FMS, while there may be many ways of reducing makespan. A possible fallout of minimizing makespan may be increased material handling and setup costs. The benefit from makespan reduction may not offset the material handling and setup costs increase. To seek a better trade-off, the makespan consideration is, therefore, incorporated to the overall production cost.

Since the secondary objective models of FMS sub-system are identical

to the ones presented in last two chapters, they are not listed here again. Also, since the secondary objective model of CMS sub-system can be applied for a pure loading problem in a 'pure' CMS system, this problem is treated separately in the next chapter. Therefore, only the primary objective model is shown below.

Model for Primary Objective

Model M-3

The objective function is

$$\text{Min. } \sum_{i=1}^I [SC_i + HC_i \Delta L] \bar{X}'_i$$

Since $\bar{X}'_i = 1 - \bar{X}_i$ in this case, the objective can be written as

$$\text{Min. } \sum_{i=1}^I [SC_i + HC_i \Delta L] (1 - \bar{X}_i)$$

or (The optimal values for the decision variables remain unchanged.)

$$\text{Max. } \sum_{i=1}^I [SC_i + HC_i \Delta L] \bar{X}_i \quad (5-1)$$

Subject to (for FMS I type sub-system)

$$\bar{X}_i - \sum_{t=1}^T \sum_{k=1}^K \bar{X}_{i,jtk} = 0 \quad \forall i$$

$$j = 1, \dots, J_i \quad (5-2)$$

$$\sum_{i=1}^I \sum_{j=1}^{J_1} \sum_{t=1}^T \bar{t}_{ijtk} \bar{X}_{ijtk} \leq A_k \quad k \in K_f \quad (5-3)$$

$$\sum_{i=1}^I \sum_{j=1}^{J_1} \bar{X}_{ijtk} - M Y_{tk} \leq 0 \quad \forall t, k \in K_f \quad (5-4)$$

$$\sum_{t=1}^T Q_t Y_{tk} \leq TQ_k \quad k \in K_f \quad (5-5)$$

$$\bar{X}_i = 0, \text{ or } 1 \quad \forall i \quad (5-6)$$

$$Y_{tk} = 0, \text{ or } 1 \quad \forall t, k \in K_f \quad (5-7)$$

$$0 \leq \bar{X}_{ijtk} \leq 1 \quad \forall i, t, k \in K_f \quad (5-8)$$

$j = 1, \dots, J_1$

or subject to (for FMS II type FMS sub-system)

$$\bar{X}_i - \sum_{t=1}^T \sum_{k=1}^K \bar{X}_{ijtk} = 0 \quad \forall i$$

$j = 1, \dots, J_1 \quad (5-2')$

$$\sum_{i=1}^I \sum_{j=1}^{J_1} \sum_{t=1}^T \bar{t}_{ijtk} \bar{X}_{ijtk} \leq A_k \quad k \in K_f \quad (5-3')$$

$$\sum_{i=1}^I \sum_{j=1}^{J_1} \bar{t}_{ijtk} \bar{X}_{ijtk} - L_t V_{tk} \leq 0 \quad \forall t, k \in K_f \quad (5-4')$$

$$\sum_{t=1}^T Q_t V_{tk} \leq TQ_k \quad k \in K_f \quad (5-5')$$

$$\bar{X}_i = 0, \text{ or } 1 \quad \forall i \quad (5-6')$$

$$V_{tk} \geq 0, \text{ integer} \quad \forall t, k \in K_f \quad (5-7')$$

$$0 \leq \bar{X}_{1,jtk} \leq 1$$

$$\forall i, t, k \in K_f \quad (5-8')$$

$$j = 1, \dots, J_i$$

The first term in the objective is setup cost and the second term is work in process holding cost if a part is allocated to CMS. It is assumed that each job -- a combination of (i, j) -- can be processed on alternative machines with different setup costs. At the beginning of the period, however, the machine to which the job will be assigned is not known. So, the average setup cost, an arithmetic mean of the setup costs on the alternative machines for a job (i, j) , is used. It is recognized that this is an approximation. It is applied because it seems difficult to find a better substitute.

Here, since part selection problem disappears, all part orders belonging to the same part type have the same importance. Therefore, subscript i (part type) is used in this model instead of p (part order). Constraints (5-2)--(5-8) correspond to constraints (3-2)--(3-8) in Chapter 3, and constraints (5-2')--(5-8') correspond to constraints (4-2) -- (4-8) in Chapter 4. It is noticed that there is no constraint related to CMS sub-system because the production capacity of CMS is assumed to be sufficient.

The value of ΔL in (5-1) can be determined based on the historical data of a company. According to a survey (Bessant and Hayward 1986) of 30 British manufacturing companies, the figures of lead time reduction are available in 23 of the 50 FMSs (FMCs). The lead time reductions vary between 40% to 92% with an average of 74%. However, in some companies the ΔL value may not be available. In this case ΔL value may

be estimated based on the reduced setup time, transportation time, and reduced machining time by using FMS instead of CMS. However, it is recognized that the estimated lead time difference may not be accurate. The effect of this inaccuracy on the decision will be discussed using an example later.

b. Model for Case 2

In this case since the production capacity is not sufficient to process all parts, the most important part orders have to be selected for immediate production. The importance is measured by a weightage factor. As described earlier, the weightage could be order quantity, dollar value of the type of part or a due date measure. This corresponds to the primary objective. An additional assumption is that the material handling capacity in the CMS portion is sufficient for all parts assigned to it.

For the reasons similar to those outlined in case 1, the secondary objective models for the FMS sub-system are not described and the secondary objective model for the CMS portion alone will be presented in the next chapter.

Model for Primary Objective

Model M-4

$$\text{Max. } \sum_{p=1}^P W_p (X_p + X'_p)$$

Subject to (if the FMS sub-system is of FMS I type)

$$X_p - \sum_{t=1}^T \sum_{k=1}^K X_{pjtk} = 0 \quad \forall p$$

$$j = 1, \dots, J_p \quad (5-2a)$$

$$\sum_{p=1}^P \sum_{j=1}^{J_p} \sum_{t=1}^T t_{pjtk} X_{pjtk} \leq A_k \quad k \in K_f \quad (5-3a)$$

$$\sum_{p=1}^P \sum_{j=1}^{J_p} X_{1jtk} - M Y_{tk} \leq 0 \quad \forall t, k \in K_f \quad (5-4a)$$

$$\sum_{t=1}^T Q_t Y_{tk} \leq TQ_k \quad k \in K_f \quad (5-5a)$$

$$X_p = 0, \text{ or } 1 \quad \forall p \quad (5-6a)$$

$$Y_{tk} = 0, \text{ or } 1 \quad \forall t, k \in K_f \quad (5-7a)$$

$$0 \leq X_{pjtk} \leq 1 \quad \forall p, t, k \in K_f \quad (5-8a)$$

$$j = 1, \dots, J_p$$

or (if the FMS sub-system is of FMS II type)

$$X_p - \sum_{t=1}^T \sum_{k=1}^K X_{pjtk} = 0 \quad \forall p$$

$$j = 1, \dots, J_p \quad (5-2'a)$$

$$\sum_{p=1}^P \sum_{j=1}^{J_p} \sum_{t=1}^T t_{pjtk} X_{pjtk} \leq A_k \quad k \in K_f \quad (5-3'a)$$

$$\sum_{p=1}^P \sum_{j=1}^{J_p} t_{pjk} X_{pjk} - L_t V_{tk} \leq 0 \quad \forall t, k \in K_f \quad (5-4'a)$$

$$\sum_{t=1}^T Q_t V_{tk} \leq TQ_k \quad k \in K_f \quad (5-5'a)$$

$$X_p = 0, \text{ or } 1 \quad \forall p \quad (5-6'a)$$

$$V_{tk} \geq 0, \text{ integer} \quad \forall t, k \in K_f \quad (5-7'a)$$

$$0 \leq X_{pjk} \leq 1 \quad \forall p, t, k \in K_f \quad (5-8'a)$$

$$j = 1, \dots, J_p$$

and

$$X'_p - \sum_{k \in K_c} X'_{pjk} = 0 \quad \forall p$$

$$j = 1, \dots, J_p \quad (5-9)$$

$$\sum_{p=1}^P \sum_{j=1}^{J_p} t'_{pjk} X'_{pjk} \leq A_k \quad k \in K_c \quad (5-10)$$

$$X'_p + X_p \leq 1 \quad \forall p \quad (5-11)$$

$$X'_p = 0, 1 \quad \forall p \quad (5-12)$$

where, constraints (5-2a) -- (5-8a) and (5-2'a) -- (5-8'a) correspond to constraints (3-2) -- (3-8) and (4-2) -- (4-8), respectively. Constraints (5-9) and (5-10) represent the restrictions in CMS sub-system similar to those represented by (5-2) and (5-3) in FMS sub-system. Constraint (5-11) states that a part order is either entirely selected or entirely rejected but once it is selected it will be entirely allocated to either the CMS or the FMS.

5.3 Illustrative Examples

Example 5.1. (Example for case 1)

Consider a manufacturing system consisting of a CMS and an FMS II type FMS sub-systems. The FMS sub-system is the same as the FMS in the example of Chapter 4 with the following data:

- . the planning horizon is 125 hours;
- . the maximum utilization limit of each machining center is 80%;
- . each machining center has a magazine with 80 tool slots;
- . 15 types of tools are available;
- . there is no tool transportation device available;
- . 6 part types are to be processed in the hybrid system, each requiring 3 operations.

The tool-machine and tool-job compatibilities, tool slot requirements, tool life, machining cost and machining time data are the same as in Chapter 4 and are listed in Tables 3.1 and 3.2.

The CMS sub-system has sufficient capacity to process all parts that may not be allocated to the FMS portion. Since the production capacity of the FMS portion is limited, not all parts can be assigned to it. Therefore, the objective is to assign a sub-set of parts to the FMS sub-system such that the savings of setup costs and work in process inventory costs can be maximized. Since the setup cost for FMS is very small and is therefore negligible (Stecke, 1983; Avonts, 1988), the setup cost saving of allocating a part to FMS portion actually equals the setup cost of the part required in CMS. The savings of setup and inventory of each part, if which is assigned to FMS sub-system, are

shown in Table 5.1.

Cost saving if allocate the part to FMS (\$)	part type					
	1	2	3	4	5	6
SC_i	78	122	260	50	122	74
$HC_i \Delta L$	90	130	160	34	130	10

$$SC_i = \sum_{j=1}^{J_i} \sum_{k \in K_{ij}} R_{ijk} / K(i, j)$$

$K(i, j)$ = number of feasible machines for job (i, j) in CMS.

Table 5.1. Setup and inventory cost saving data for example 5.1.

The model M-3 is applied to this example problem. The output is summarized in Table 5.2.a. As can be seen in Table 5.2.a., parts 2--5 are assigned to the FMS sub-system and parts 1 and 6 are allocated to the CMS segment. The operation allocation, tool selection and assignment decisions can also be made based on the output for the FMS. If some secondary objectives are desired, model M2-1 or M2-2 can be used. The procedures are the same as described in the previous chapter. For the CMS portion, the operation allocation (machine loading) decision procedure will be addressed in the next chapter.

Resources in FMS sub-system: $TQ_1=TQ_2=TQ_3=80$, $A_1=A_2=A_3=100$ hours
Part types assigned to FMS sub-system: 2,3,4,5
Part types assigned to CMS sub-system: 1,6
Cost saving: \$1,008

Operation and tool allocation pattern in FMS sub-system

Machining center	Job assignment (%)	Tool assignment
	(1)	(2)
M1	J31/100, J33/58, J41/70, J43/100, J51/100, J53/14	T1/4, T2/5, T4/4, T7/7, T11/14
M2	J21/4, J32/30, J33/42, J41/30, J42/100, J52/100, J53/86	T1/1, T6/12, T8/10, T10/5, T14/7
M3	J21/96, J22/100, J23/100, J32/70	T3/8, T5/11, T7/2, T9/3, T15/3
Process routes:	part 2: $\begin{matrix} M3 \\ M2 \end{matrix} \} - M3 - M3, \quad (3)$ part 4: $\begin{matrix} M1 \\ M2 \end{matrix} \} - M2 - M1,$	part 3: $M1 - \begin{matrix} M3 \\ M2 \end{matrix} \} - \begin{matrix} M1 \\ M2 \end{matrix}$ part 5: $M1 - M2 - \begin{matrix} M2 \\ M1 \end{matrix}$

(1) J31/100 means 100% of job J31.

(2) T1/4 means 4 tools of tool type 1.

(3) Indicates that the operation 1,2 and 3 of part 2 are assigned to machining center M3 and M2, M3, and M3, respectively.

Table 5.2.a. Solution for example 5.1.

As mentioned earlier, the ΔL value may not be available in some companies and may be obtained based on estimation. The effect of the inaccuracy of the ΔL value on the decision process is discussed

through sensitivity analysis of ΔL for the above example problem. The result is shown in Table 5.2.b. According to the survey conducted by Bessant and Hayward (1986), the lead time reductions vary between 40% to 92%. Our sensitivity analysis is, therefore, performed within the range of 40% to 90%.

Lead time ⁽¹⁾ reduction b (%)	part type	$SC_1 + HC_1 \Delta L$ (\$)						Part types allocated to FMS sub- system
		1= 1	2	3	4	5	6	
40		138	209	367	73	209	81	2, 3, 4, 5
50		168	252	420	80	252	84	2, 3, 4, 5
60		213	317	500	101	317	89	2, 3, 4, 5
70		288	425	633	129	425	97	2, 3, 4, 5
80		438	642	900	186	642	114	2, 3, 4, 5
90		888	1292	1700	356	1292	164	2, 3, 4, 5

(1) b is the percentage of lead time reduction. The relationship between b and ΔL is $\Delta L = b \cdot l$, where, $l = L / (1 - b)$, l is the lead time for CMS and L is the lead time for FMS, here L is assumed to be the same as A (planning horizon of FMS).

Table 5.2.b. Sensitivity analysis of lead time reduction (for example 5.1)

Table 5.2.b. shows that the load sharing decision is not sensitive to the percentage of lead time reduction b (and hence, ΔL) within the range of 40% to 90%. This indicates that if other parameters, e.g., SC_1 and HC_1 , are fixed, the load sharing decision made based on a

rough estimate of lead time difference between CMS and FMS sub-systems is appropriate enough for practical applications.

Example 5.2. (Example for case 2)

In this example, the FMS sub-system is the same as in example 5.1 and the associated data of the FMS sub-system are shown in Tables 3.1 and 3.2. The CMS sub-system has 3 machines, each machine has 100 hours processing time available. 8 types of candidate parts (each has only one part order) are considered for processing in the hybrid system. The machining time data in FMS and CMS sub-systems are listed in Table 5.3. The market prices of the 8 types of parts are: \$20,000, \$30,000, \$50,000, \$10,000, \$30,000, \$10,000, \$10,000 and \$10,000, respectively. The production capacities of both sub-systems are not enough to process the 8 types of parts. The primary objective is to select a sub-set of parts for processing in both sub-systems such that the combined capacity can be best utilized to maximize the dollar output.

Model M-4 is used for this example. The solution is shown in Table 5.4. Part 1,2 and 4 are assigned to the FMS sub-system, part 3 and 6 to CMS sub-system. part 5,7 and 8 are rejected and left for future processing. The machine loading decision in FMS portion can also be made based on the solution if no secondary objectives are considered. The loading decision in CMS portion will be discussed in the following chapter.

Compatible tools		Processing time(hrs)					
		in FMS			in CMS		
Job		MC1	MC2	MC3	MC4	MC5	MC6
J11	2	25			25	30	
	4	20	20				
	14		18				
J12	1	10	15				40
	2	10					
	13	12		15			
J13	9			10			40
	10		20				
	15			12			
J21	3	30		25	60		
	6		35				
	8		40				
J22	5			20		50	
	7	18		20			
J23	8		10		30		
	9			6			
J31	1	45	40			50	
	3	45		40			
	11	40					
J32	5			15		40	
	10		20				
	12		20				
J33	6		32		40		40
	7	25		30			
	14		38				

Table 5.3. Machining time data for example 5.2.

Table 5.3. continued

		Processing time(hrs)					
		In FMS			In CMS		
Job		MC1	MC2	MC3	MC4	MC5	MC6
J41	1	8	10			25	
J42	3	10		20	25		
	10		9				
J43	4	12	12				35
J51	2	10			20		
J52	8		30			40	
J53	6		40				60
	7	45		50			
J61	4	5	8		15	20	
	8		5				
J62	9			10			20
J63	10		15		20		
J71	9		50		90		
J72	12			55			95
J81	14	50			85		
J82	2			60			90

Table 5.3. Machining time data for example 5.2. (Continued)

Resources in FMS sub-system: $TQ_1=TQ_2=TQ_3=80$, $A_1=A_2=A_3=100$ hours
Resources in CMS sub-system: $A_4=A_5=A_6=100$ hours
Part types assigned to FMS sub-system: 1,2,4
Part types assigned to CMS sub-system: 3,6
Rejected part types: 5,7,8

Operation and tool allocation pattern in FMS sub-system

Machining center	Job assignment (%)	Tool assignment
M1	J11/100, J12/80, J22/83, J41/100, J43/100	T1/4, T2/4, T4/9, T7/5, T13/1
M2	J13/70, J21/92, J23/100	T6/5, T8/10, T10/3, T14/2, T9/14
M3	J12/20, J13/30, J21/7, J22/17, J42/100	T3/8, T5/5, T7/6, T9/5, T13/1, T15/1

Process routes: part 1: M1 -{ $\begin{matrix} M1 \\ M3 \end{matrix}$ }- { $\begin{matrix} M2 \\ M3 \end{matrix}$ }, part 2: $\begin{matrix} M2 \\ M3 \end{matrix}$ }- { $\begin{matrix} M1 \\ M3 \end{matrix}$ }- M2

part 4: M1 - M3 - M1

Table 5.4. Solution for example 5.2.

5.4 Solution Methodology

5.4.1 Solution Procedure for Model M-3

The structure of model M-3 is the same as model M-1 (for a FMS I type sub-system) or model M-2 (for a FMS II type sub-system). Therefore, by substituting W_p with $(SC_1 + HC_1 \Delta L)$ in the solution procedures of model M-1 or M-2 the solution procedure for model M-3 is obtained.

5.4.2 Solution Procedure for Model M-4

The main difficulty in solving model M-4 arises from constraint (5-11). If constraint (5-11) can be dropped, model M-4 becomes two independent subproblems as follows:

Sub-problem 1 (associated with FMS sub-system)

Model M4-1

$$\text{Max. } \sum_p W_p X_p$$

subject to (5-2)--(5-8), or (5-2')--(5-8'), depending on the type of FMS sub-system.

Sub-problem 2 (associated with CMS sub-system)

Model M4-2

$$\text{Max. } \sum_p W_p X'_p$$

subject to (5-9) and (5-10).

Constraint (5-11) is removed by an approximate manipulation. Instead of directly solving model M-4, model M4-1 is first solved using the same procedure for solving model M-1 or M-2 as described in Chapter 3 and 4. After solving model M4-1, a sub-set of part orders, denoting as Ψ_f , is selected from all candidate part orders (the associated set is denoted by Ψ) for the FMS sub-system. Part orders for CMS sub-system are then selected from the remaining part order set $\Psi - \Psi_f$ by solving model M4-2 with a restriction of $pe(\Psi - \Psi_f)$. Obviously, the solution obtained based on this manipulation will satisfy constraint (5-11).

Model M4-2 has a relatively small size and can be easily solved using existing packages, or using a similar procedure for solving sub1-1 or sub2-1 as shown in Chapter 3 or 4.

CHAPTER VI

THE MACHINE LOADING AND PROCESS ROUTE SELECTION PROBLEM IN A CONVENTIONAL MANUFACTURING ENVIRONMENT

This chapter addresses the machine loading and process route selection problem in a conventional manufacturing environment, which could be a 'pure' conventional manufacturing system(CMS), e.g., a job shop or a mass production system with a process layout, or a CMS sub-system in a hybrid system as discussed in Chapter 5. The problem background and scenario are described in the next section.

6.1 Problem Description

The machine loading problem in flexible manufacturing systems (FMSs) has been modeled and solved by a large number of researchers (Stecke and Solberg, 1981; Stecke, 1983; Kusiak, 1983; Chakravarty and Shtub, 1984; O'Grady and Menon, 1987; Lashkari *et al.* 1987; Kumar *et al.* 1987; and Sarin and Chen, 1987). For CMS , however, the loading problem has not caught enough attention in literature. This may be mainly due to the fact that, in conventional manufacturing environments, researchers simply assume that for any part only one process plan is available, i.e., any operation of a part can be assigned to only one predetermined machine. The consequence is that each part has a fixed route through the manufacturing system. Therefore, it seems as though no loading problem exists. In practice, however, it is not unusual that one operation can be assigned to several alternative machines. That is why CMS is flexible. Today,

there are still many conventional manufacturing systems in use. Even in some FMSs, the functional manufacturing facility and flexible manufacturing cells (FMCs) may simultaneously exist so that some parts which could not be processed by the FMCs can be assigned to the functional facility (Hutchinson and Holland, 1982; Kusiak, 1987; Avonts *et al.*, 1988; Gupta *et al.*, 1988; and Kayaligil, 1990). So, from both an academic and/or practical point of view, it is very important to explore the machine loading problem in a conventional manufacturing environment.

Process planning, both in advanced or conventional manufacturing systems, has been thought of as the link between engineering design and shop floor manufacturing and is a major determinant of manufacturing cost and profitability. Recently, although many automatic process planning systems (APPSs) have been developed, "The degree of automation for many of the existing PPSs is rather low and requires a considerable proportion of human interaction. It is difficult to assess the quality of process plans... . However, due to the high complexity of the problem, it is possible to assert that the quality of process plans provided by the existing PPSs could be significantly improved." (Kusiak, 1985a). Obviously, it is also possible to significantly improve the process plans provided by manual methods.

To improve the quality of process plans, Kusiak (1985a) proposed an integer programming approach to find an optimum process plan by means of computing a set of optimum tool paths in terms of minimum sum of

tool path costs and then ordering them by the topological order algorithm based on certain technological constraints. Szadkowski (1971) presented a procedure applying graph theory and Bellman's optimum principle to solve the machining process optimization problem. Both these papers considered the process optimization problem for only one type of part and did not include the loading problem in the analysis. In the real world, no matter whether FMSs or conventional manufacturing systems are being investigated, the usual case is that many parts are processed simultaneously in a system. Under this situation, competition for resources related to machining is inevitable. Therefore, an optimal process plan obtained by considering only one part each time (i.e., considering each part by separating it from others) may not be the best one; furthermore, no material handling cost is considered in these papers and even in other literature on loading problems (Kusiak, 1985 b; Sarin and Chen, 1987). The models may therefore be inadequate.

Most previous researchers have assumed that no batch splitting occurs, i.e., one operation can be assigned to only one machine, which may not be the case in real world situations. Under certain circumstance, it is necessary to assign an operation to more than one machine to balance machine utilization, reduce makespan or to realize some other practical requirements. In this case, attention should be paid not only to operation assignment but also to the proportion of the operation to be assigned to each feasible machine. It follows that the process planning (process route selection) and the machine loading problem should be taken into account simultaneously. Due to the above

reasons, a new approach or model is needed to deal with both machine loading and process planning problems in a conventional manufacturing environment.

Here, we consider a CMS with K non-identical machines, each located at a fixed location. I types of parts are to be processed in batches within one period. Each part needs one or more operations and each operation can be performed on one or more machines with different machining and setup times and are related to different material handling costs. Any operation on a batch of the same parts may be split into two or more sub-batches, i.e., one operation may be simultaneously performed on different machines. The purpose is to determine the optimum machine load (work content) for each machine and optimum process plan (or process route) for each part in terms of minimum production cost (including machining cost, setup cost, material handling cost, machine idle cost and the penalty towards long makespan) under the following assumptions:

- (1) The compatibility of each machine-job combination is known.
- (2) The data related to machining and setup times of different operations at different machines, the costs per unit time of machining, setup and idle time of each machine as well as material handling cost per part per unit distance are given.

In this chapter, the following notations are used.

Subscripts

$i = 1, \dots, I$ part type

$J = 1, \dots, J_i$ operation

$k, l = 1, \dots, K$ machine (or work center)

Variables

V'_{ijk} = a 0-1 integer variable, if operation $j-1$ of part i is not allocated to machine k but operation j of part i is allocated to machine k , then $V'_{ijk} = 1$; otherwise, $V'_{ijk} = 0$.

X'_{ijkl} = number of components of part type i undergoing operation j assigned to machine k and to be transported to machine l ($j=0$ and $k=0$ indicate a dummy operation, i.e., the transportation from raw material storage; $l=K+1$ means a location other than the k locations where the K machines are located -- may be a final production storage place or a assembly department).

Y'_{ijk} = a 0-1 integer variable; if operation j of part i is assigned to machine k , $Y'_{ijk} = 1$; otherwise, 0.

Parameters

b_i = material handling cost per unit part per unit distance

C = overall system running cost per unit time (except material handling equipment running cost)

C'_{ijk} = machining cost(per unit part) of operation j of part i at machine k , $C'_{ijk} = t'_{ijk} C_k$

C_k = machining cost per unit time on machine k

d_{kl} = distance from machine k to l

D_i = the demand for part type i

H'_{ikl} = material handling cost per unit part i from machine k to l
 $(H'_{ikl} = b_i * d_{kl})$

r = proportion of setup cost within total production cost

R_{ijk} = the cost per major setup for operation j of part i at machine k, $R_{ijk} = R'_{ijk} * R_k$

R'_{ijk} = the time needed per major setup for operation j of part i at machine k

R_k = major setup cost per unit time on machine k

S_{ijk} = the cost per minor setup for operation j of part i at machine k, $S_{ijk} = S'_{ijk} * S_k$

S'_{ijk} = the time needed per minor setup for operation j of part i at machine k

S_k = minor setup cost per unit time on machine k

t'_{ijk} = the time needed to process one unit part type i of operation j at machine k

Set

I' = a set of part types to be processed in the CMS system

Definitions of major and minor setups are provided in the next section.

6.2 Model Formulation

In machine loading problems, all the selected part orders have to be produced simultaneously. Therefore if several selected part orders belong to one part type, their importance or priority will be the same. There is no need to consider them separately and hence the variables should be defined in terms of part type rather than part order to simplify the model and solution procedure.

a. Formulation of the Constraints

Two types of constraints are defined as follows:

(1) Input/output equivalence constraints

For any operation of any part, the number of components which are input to a machine should be equal to the output of components from the machine (Note: no scrap is considered here). Further the total number of components of any part input to the system should be equal to the predetermined requirement. These relationships can be expressed as follows (For convenience, the set of part types to be processed in the system is denoted by I'):

$$\sum_{k=1}^K X'_{100k} = D_i \quad i \in I'$$

$$X'_{100k} - \sum_{q=1}^K X'_{11kq} = 0 \quad i \in I', \forall k$$

$$\sum_{q=1}^K X'_{1j-1qk} - \sum_{l=1}^K X'_{1jkl} = 0 \quad i \in I', \forall k \quad (6-1)$$

$$j=1, 2, \dots, J_i-1$$

$$-X'_{1J_i kK+1} + \sum_{q=1}^K X'_{1J_i-1qk} = 0 \quad i \in I', \forall k$$

$$\sum_{k=1}^K X'_{1J_i kK+1} = D_i \quad i \in I'$$

(2) Setup constraints

Two types of setups are considered, i.e., minor and major setups. A minor setup is one in which a changeover from one operation to another on the same part type takes place on the same machine. A minor setup

can be expressed by using an indicator variable Y'_{ijk} , such that

$$\sum_{l=1}^K X'_{ljk1} \leq D_i Y'_{ijk} \quad i \in I', \forall k \quad (6-2a)$$

$$j=1,2,\dots,J_i-1$$

$$X'_{iJ_i kK+1} \leq D_i Y'_{iJ_i k} \quad i \in I', \forall k \quad (6-2b)$$

For the final operation of any part, the components will be assigned to a storage warehouse or a assembly department rather than any machines in the system, therefore, a separate constraint is required as shown in (6-2b).

A major setup occurs only when the part type to be processed on the machine is different from the one just completed. Usually, a major setup needs more time and hence has a higher cost than minor setup. A 0-1 variable V'_{ijk} is used to indicate the major setups and should satisfy the following conditions:

$$Y'_{ijk} - Y'_{ij-1k} \leq V'_{ijk} \quad (6-3)$$

$$Y'_{i0k} = 0$$

$$i \in I', j=1,2,\dots,J_i, \forall k$$

It is interesting to note the following. If it is assumed that an operation can be assigned to only one machine and if only the major setup and material handling cost are the main concerns, the machine load and process route can be determined by simply solving the following model:

$$\text{Min } Z' = \sum_{i \in I'} \sum_{j=1}^{J_1} \sum_{k=1}^K V'_{ijk} \quad (6-4)$$

subject to constraints expressed by equations (6-1)-(6-3).

This is because Z' indicates not only the number of major setups but also the number of component movements between machines. It is noticed that a similar objective function in non-linear form was developed by Stecké (1983). This objective function can be only applied under the same assumption, i.e., a operation can be assigned to only one machine.

b. Formulation of the Objective Function

The objective function consists of 5 different costs, which represent the major contributors to the overall production cost.

(1) Machining cost

Based on assumption (2), the machining cost can be expressed as:

$$Z_1 = \sum_{i \in I'} \sum_{j=1}^{J_1-1} \sum_{k=1}^K \sum_{l=1}^K C'_{ijkl} X'_{ijkl} + \sum_{i \in I'} \sum_{k=1}^K C'_{ij_1k} X'_{ij_1kK+1} \quad (6-5)$$

The machining cost for the final operation is expressed separately due to the same reason as indicated for constraint (6-2). For the infeasible operation-machine combinations, let $C'_{pjk} = M$, a large positive value.

(2) Material handling cost

$$\begin{aligned}
 Z_2 = & \sum_{i \in I'} \sum_{j=1}^{J_1-1} \sum_{k=1}^K \sum_{l=1}^K H'_{ikl} X'_{ijkl} + \sum_{i \in I'} \sum_{k=1}^K H'_{i01} X'_{i01} \\
 & + \sum_{i \in I'} \sum_{k=1}^K H'_{ikK+1} X'_{iJ_1 kK+1}
 \end{aligned} \quad (6-6)$$

The last two terms represent the cost of transporting raw material from storage to the system and the cost of shipping final products to the warehouse or assembly department, respectively.

(3) Setup cost

$$Z_3 = \sum_{i \in I'} \sum_{j=1}^{J_1} \sum_{k=1}^K (S_{ijk} Y'_{ijk} + R_{ijk} V'_{ijk}) \quad (6-7)$$

The two terms are minor and major setup costs, respectively.

(4) Machine idle cost

This idle cost includes both machinery and labor idle cost due to unbalanced loads.

$$Z_4 = \sum_{k=1}^K C_k (B - B_k) \quad (6-8a)$$

$$\text{where, } B_k = \sum_{i \in I'} \sum_{j=1}^{J_1-1} t'_{ijk} \sum_{l=1}^K X'_{ijkl} + \sum_{i \in I'} t'_{iJ_1 k} X'_{iJ_1 kK+1}$$

$$+ \sum_{i \in I'} \sum_{j=1}^{J_1-1} (S'_{ijk} Y'_{ijk} + R'_{ijk} V'_{ijk}) \quad (6-8b)$$

$$B = \max_{1 \leq k \leq K} \{B_k\} \quad (6-8c)$$

It should be noted that in most previous researches this cost was replaced by using a machine availability constraint. Obviously this manipulation is simple but if the predetermined machine availability factor (or system efficiency factor) is high it may result in high unbalance and longer makespan.

(5) The penalty towards long makespan

The purpose of this penalty cost is to attain shorter makespans; hence the work in process cost can be partially offset by reducing makespan. Since a functional layout manufacturing system is virtually a general job shop, it is very difficult to accurately schedule the system in terms of minimizing makespan. So the following approximate method is applied.

First an upper bound of makespan T_u is proposed, which is defined by:

$$T_u = \sum_{j=1}^{J_1-1} t'_j \quad (6-9a)$$

$$\text{where } t'_j = \max_{1 \leq k \leq K} \{t'_{kj}\} \quad (6-9b)$$

$$t'_{kj} = \sum_{i \in \Phi(1 | J_1-1 \geq 1)} (t'_{ijk} \sum_{l=1}^K X'_{ijk1} + S'_{ijk} Y'_{ijk} + R'_{ijk} V'_{ijk})$$

$$\forall k, j=1,2,\dots,J_1-1 \quad (6-9c)$$

$$\text{and } t'_{k,j_1} = t'_{1,j_1,k} X'_{1,j_1,k,k+1} + S'_{1,j_1,k} Y'_{1,j_1,k} + R'_{1,j_1,k} V'_{1,j_1,k} \quad (6-9d)$$

$$\forall k$$

This upper bound may be illustrated by an example. Suppose there is a 3 machine manufacturing system in which 2 parts are to be produced. Part 1 needs 3 operations and part 2, 2 operations. If the system is loaded as shown in Figure 6.1 (the values above each arc are the total machining time and setup time of the operation on the related machine) then

$$t'_1 = \max \{t'_{k1}\} = \max \{100+50, 50, 150\} = 150$$

$$t'_2 = \max \{t'_{k2}\} = \max \{120+50+70, 0, 120\} = 240$$

$$t'_3 = \max \{t'_{k3}\} = \max \{0, 210, 0\} = 210$$

The upper bound of the makespan is: $T_u = t'_1 + t'_2 + t'_3 = 600$

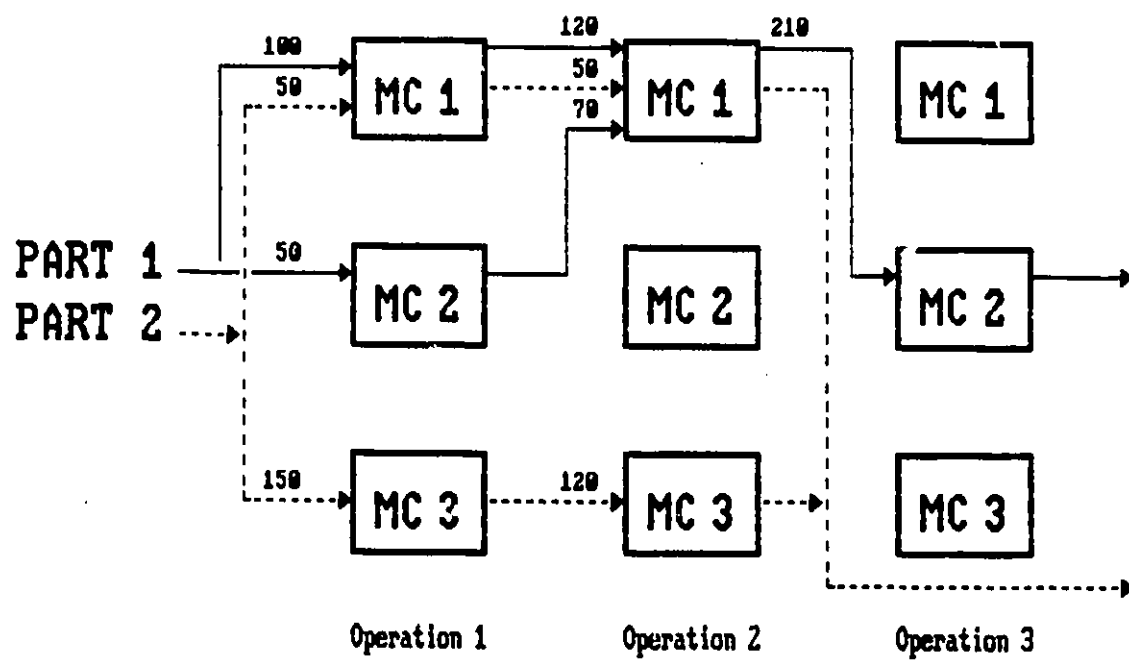


Figure 6.1. An illustration of the upper bound of makespan

The approximate minimum makespan T_u^* can be obtained by minimizing (6-9a) subject to conditions (6-1)-(6-3). But this may cause all jobs assigned to all feasible machines to be finished at the same time and will result in more batch splittings, and consequently, more material handling movements, setups and high material handling cost and setup costs. To avoid this, allow the makespan to be slightly greater than T_u^* . This is realized by penalizing the time difference between T_u and T_u^* . This penalty value should reflect the opportunity cost. For this purpose a penalty cost per unit time is imposed for longer makespan. It follows that:

$$Z_5 = C(T_u - T_u^*) \quad (6-10)$$

Therefore the overall model would be

Model M-5

$$\text{Min } Z = \sum_{m=1}^5 Z_m \quad (6-11)$$

subject to constraints shown in equations (6-1)-(6-3).

6.3 Illustrative Example

Example 6.1.

Consider a small system with three machines. Two parts are to be produced within a period and each part needs three operations. The related data are shown in Tables 6.1--6.3. Assume that the material

handling costs per unit distance per component are \$0.5 and \$1 for part 1 and 2 respectively.

The model for the example problem contains 67 constraints, 42 continuous variables and 36 0-1 integer variables. By deleting the infeasible operation-machine combinations, the model reduces to 51 constraints, 16 continuous variables and 22 0-1 integer variables. In order to minimize Z , first compute $T_u^* = 80.68$ by minimizing (6-9a). Then, with this T_u^* value solve model MD1 and obtain the final result (see Table 6.4). The model was solved using the LINDO micro-computer package. Each run required less than 8 minutes for T_u^* and 4 minutes for Z on a IBM ATPC machine.

To gain some further insight into the problem, the following issues are addressed by means of additional computations.

a. The Impact of Splitting on Cost and Makespan

The effect of splitting on cost and makespan was investigated through a comparison of the situation where splittings (SP) are allowed with one where no splitting (NSP) is allowed. To do so a new model should be formed by adding an additional constraint which restricts each operation to only one machine. The new model is shown as follows:

Model M-6

$$\text{Min } Z = \sum_{m=1}^5 Z_m$$

subject to (6-1)-(6-3), and

$$\sum_{k=1}^K Y_{ijk} = 1 \quad i \in I', \quad j=1,2,\dots,J_1$$

The outputs for both situations are summarized in Table 6.5. The table shows that the optimal process route and machine load are very different in the two cases. In the NSP case the overall cost is \$2370 higher and the makespan is 20 hours (about 1/5 of total makespan) longer than in SP case. So SP is preferred to NSP.

The following two sections will refer to Table 6.6 which was obtained by solving equation (6-11) with various demand patterns.

b. The Effect of the Proportion (r) of Setup Cost within the Total Cost

Referring to Table 6.6, it is found that the process choice of routes is significantly influenced by the proportion of setup cost in the overall cost; other costs do not have similar effects on routes since they vary in almost the same proportion as demands. When r varies between 0.1603 and 0.2916, there is no batch splitting for either part on any operation; when $r=0.0848$ --0.1281, part 1 does not split but part 2 splits at operation 2; when $r=0.0602$, both parts on operation 2 have split batches; when $r \leq 0.0514$, there is no batch splitting for part 1 but part 2 is split twice during operation 1 and 2.

c. The Effect of Demands

From Table 6.6 it is observed that lower production volume (demand)

corresponds to higher r value. This implies that for low production volumes (namely, low demand) NSP is preferred and hence the assumption that each operation can be assigned to only one machine is appropriate. On the other hand, higher demand corresponds to lower r value, i.e., lower setup cost proportion, so SP may be preferred to obtain shorter makespan; consequently the assumption is not appropriate for medium or large size demand cases.

Part type	Operation	Machine	Machining per component	Time needed per minor setup (hrs.)	Time needed per major setup (hrs.)
1	1	1	0.15	0.30	2.00
		2	0.10	0.20	1.00
		3			
	2	1	0.15	0.30	2.00
		2			
		3	0.10	0.20	1.00
	3	1			
		2	0.15	0.20	1.00
		3	0.10	0.20	1.00
2	1	1	0.20	0.30	2.00
		2	0.15	0.20	1.00
		3			
	2	1			
		2	0.20	0.20	1.00
		3	0.15	0.20	1.00
	3	1	0.15	0.30	2.00
		2			
		3			

Table 6.1. Machining and setup time data for example 6.1.

Cost	Machine		
	1	2	3
Machining cost/hr.	100	120	180
Minor or major setup cost/hr.	80	110	160

Table 6.2. Machining cost data for example 6.1.

From machine	To machine				
	I	1	2	3	O
I	0	5	7	5	10
1		0	2	1	5
2			0	2	3
3				0	5
O					0

Note: I and O are dummy machines which represent the raw material storage place and final parts warehouse, respectively

Table 6.3. The distance data between each pair of machines

Part type	Operation	Number of parts processed on		
		Machine 1	Machine 2	Machine 3
1	1		150	
	2			150
	3			150
2	1	114	86	
	2		135	65
	3			200

Machining cost $Z_1 = \$19024$

Material handling cost $Z_2 = \$3720$

Setup cost $Z_3 = \$210$

Idle cost $Z_4 = \$2260$

Penalty cost $Z_5 = \$860$

Maximum makespan $T = 89.74$ hours.

Table 6.4. Solution summary of example 6.4.

Operation allocation pattern							
Part type	Operation	Splitting allowed (SP)			No splitting allowed (NSP)		
		MC1	MC2	MC3	MC1	MC2	MC3
1	1		150			150	
	2			150	150		
	3			150			150
2	1	119	81			200	
	2		143	57			200
	3	200			200		
System performance							
Splitting allowed				No splitting allowed			
Total cost Z(\$)		30516		32886			
Machining cost Z ₁ (\$)		20088		19290			
MH cost Z ₂ (\$)		3737		4125			
Setup cost Z ₃ (\$)		1070		1016			
Idle cost Z ₄ (\$)		1700		1975			
Penalty cost Z ₅ (\$)		3920		6480			
Maximum makespan T (hrs)		90.72		110.90			

Table 6.5. Comparison of SP and NSP

Part type	Operation	$D_1=5$ $D_2=7$						15			30			50		
		M1	M2	M3	M1	M2	M3	M1	M2	M3	M1	M2	M3	M1	M2	M3
1	1	5			15			30						50		
	2			5			15			30			30		50	
	3			5			15			30			30		50	
2	1		7			20				40				67		
	2		7			20				34			6	52		15
	3	7			20			40			67					
Z_1			690			2013				4055				6774		
Z_2			126			363				731				1228		
Z_3			746			746				938				938		
Z_4			825			873				749				825		
Z_5			171			660				857				1296		
$Z_{1-4} = \sum_{i=1}^4 Z_i$			2387			3994				6463				9765		
$Z = \sum_{i=1}^5 Z_i$			2558			4654				7321				11061		
T			8.20			14.05				22.44				34.23		
$t (= Z_1/Z)$			0.2916			0.1603				0.1281				0.0848		

Table 6.6 The effects of changing r and D_i

Part type	Operation	75			100			150			1500		
		M1	M2	M3	M1	M2	M3	M1	M2	M3	M1	M2	M3
1	1	75				100		150			1500		15000
	2			74		100	100	150		1500		15000	
	3			75		100	100	150		1500		15000	
2	1		100		73	54	119	81	1290	790	12120	7880	
	2		75	25	133	98	35	143	57	1378	622	13721	6279
	3	100					200	2000			20000		
Z_1		10135			13365			20088		201087		2011079	
Z_2		1838			2491			3737		37255		372441	
Z_3		962			1070			1070		1070		1070	
Z_4		1050			1254			1700		13502		132623	
Z_5		1982			2620			3920		38867		388640	
$Z_{1..5} = \sum_{i=1}^5 Z_i$		13985			18180			26595		253015		2517213	
$Z = \sum_{i=1}^5 Z_i$		15967			20799			30516		291882		2905844	
T		48.60			62.52			90.72		849.00		8431.00	
$r (= Z_3/Z)$		0.0602			0.0514			0.0551		0.0037		0.00037	

Note: M1, 2, and 3 represent Machine 1, 2 and 3, respectively.

Table 6.6 The effects of changing r and D_i (continued)

CHAPTER VII

COMPUTATIONAL EXPERIENCE

Since the structures and solution procedures of the models developed in Chapters 3--4 are similar, model M-1 only has been taken as example to demonstrate the computational implementation of the solution procedures.

The algorithm A-1, heuristic Feas-1 and revised simplex method for the decomposed problems as well as the associated column generation procedures were coded using FORTRAN 77. The FORTRAN program was tested on a IBM 4381 machine (CMS operating system). Six sets of randomly generated problems were tested (Table 7.1). For all test problems, the scheduling flexibility index, the tightness of available processing time, and the tightness of magazine capacity are specified as follows:

(1) Scheduling flexibility index (e_1), which is measured by

$$e_1 = n_{pj} / T * K$$

where n_{pj} = number of alternative (t,k) combinations which can perform job (p,j), T is the total number of tool types and K is the total number of machining centers.

(2) The tightness of available processing time,

$$e_2 = \sum_k A_k / \sum_p \sum_j t_{pjtk}^l$$

where, $t_{pjtk}^l = \min_{t,k} \{ t_{pjtk} \}$, to simplify computation we set all

$t_{pjtk}^l = t^l$, and t^l is the lower limit of t_{pjtk} ($=U [t^l, t^u]$), A_k is the available machining time on machining center k .

(3) The tightness of magazine capacity,

$$e_3 = TQ_k / \sum_t Q_t$$

where TQ_k is the magazine capacity of machining center k , Q_t is the number of slots required by tool type t .

For all test problems, we set $e_1 = 0.12$, $e_2 = e_3 = 0.5$.

According to Ito (1981), the best policy for investigating an FMS is to include five machining centers. So, the number of machining centers are set to five in the computational study.

Other parameters are generated uniformly within the following ranges:

$$W_p = U [10, 100]$$

$$J_p = U [1, 5]$$

$$Q_t = U [1, 3]$$

$$t_{pjt_k} = U [2, 8](\text{hours})$$

The termination condition is set as : the maximum number of iterations is 30 or acceptable percentage error is within 5%, whichever is reached first. The percentage error equals $(ZU - ZL) * 100\% / ZU$, where, ZU is the lowest upper bound and ZL is the greatest feasible solution (lower bound).

The outputs for the 6 sets of test problems are summarized in Table 7.2. It is shown that all of test problems reach acceptable percentage error within reasonable CPU time.

Problem set	P	T	N ₁	N ₂ /N _{2a}	N ₃ /N _{3a}
1	10	20	110	5000/2500	160/135
2	10	50	260	12500/6250	310/285
3	20	20	120	10000/5000	210/160
4	20	50	270	25000/12500	360/310
5	50	20	150	25000/12500	260/235
6	50	50	300	62500/31250	510/385

N₁ -- number of integer variables

N₂/N_{2a} -- maximum/average number of continuous variables

N₃/N_{3a} -- maximum/average number of constraints

For all test problems, the number of machines is set to 5.

Table 7.1. Test problem design

Problem set	problem number	ZU	ZL	$\epsilon\%$	CPU (seconds)
1	1	404	393	2.77	4.90
	2	361	351	2.86	5.83
	3	309	300	3.06	4.99
	4	476	463	2.65	18.05
	5	354	346	2.20	15.83
Average					9.92
2	1	400	380	5.00	49.63
	2	433	428	1.16	15.70
	3	385	367	4.69	18.95
	4	321	308	4.04	16.42
	5	328	312	4.89	37.10
Average					27.56
3	1	646	634	1.81	16.74
	2	739	717	2.94	19.29
	3	705	688	2.38	20.14
	4	685	680	0.70	13.08
	5	768	758	1.32	17.14
Average					17.28
4	1	799	787	1.46	41.53
	2	625	606	3.06	38.95
	3	784	769	1.97	56.89
	4	812	795	2.05	47.75
	5	631	616	2.42	52.04
Average					47.43
5	1	1948	1846	3.83	80.36
	2	1904	1812	4.85	108.15
	3	1758	1678	4.55	337.36
	4	1719	1670	2.85	95.16
	5	1732	1669	3.63	333.80
Average					190.97
6	1	2091	2058	1.58	192.44
	2	2197	2134	2.87	246.73
	3	2105	2079	1.23	201.54
	4	1986	1919	3.37	243.65
	5	2080	2014	3.17	214.51
Average					219.77

Note: $\epsilon \% = 100\% \cdot (ZU - ZL) / ZU$, ZU and ZL are upper bound and lower bound respectively.

Table 7.2. Computational results of the test problems

CHAPTER VIII

DISCUSSIONS, CONCLUSIONS AND SUGGESTIONS FOR FUTURE RESEARCH

8.1 Discussions

(1) *Problem Scope*

As reviewed in Chapter 2, many previous researchers separately investigated the PS and ML problems. Few researchers attacked the combined PS and ML problem. Those who did investigated either only a partially combined PS and ML problem or a fully combined PS and ML problem with some many-to-one relationship assumptions, which would restrict the utilization of inherent productivity and flexibility of a FMS.

For the hybrid system, most researchers focus on the load sharing between the CMS and FMS sub-systems. The joint consideration of part selection (for the case where production capacity is limited), load sharing and machine loading has not been addressed in literature.

In view of the drawbacks in the previous researches, the full combined PS and ML problems were investigated in the present research, which dropped the many-to-one assumption for three classes of FMSs (in the CMS sub-system of the hybrid system, different manipulation is performed). The better utilization of the productivity and flexibility of FMSs can be expected. The joint treatment of part selection, load sharing and machine loading problem has also been carried out in this

research.

(2) Model Size

The difficulty in solving a model depends greatly on the number of integer variables included in the model. One of the features of the models developed for the combined PS and ML in this research is that fewer integer variables are used. For example, the number of 0-1 integer variable in Model M-1 is $P+TK$. For a problem of 20 part orders, 20 types of tools and 5 machining centers, the number of 0-1 integer variables is 120. A problem of this size can be solved using many existing packages. Some daily PS and ML decision problems may fall into this size range.

(3) Computation Efficiency of the Developed Algorithms

For larger sized problems, the algorithms developed in Chapter 3 and 4 can be applied. The algorithms were developed to provide an aid for the manager's daily or weekly PS and ML decision making. The emphasis, therefore, is on computational efficiency. The computational efficiency of one (the algorithm for Model M-1) of the developed algorithms has been demonstrated. For a problem with a size of 50 part orders, 50 types of tools and 5 machining centers, the solution with a maximum acceptable error less than 5 % can be obtained in 3--4 minutes of CPU time on a IBM 4381 machine under CMS operating system. This would be acceptable for many practical PS and ML problems.

(4) Solution Consistency

Solution consistency for both PS and ML problems is one of the

motivations for this research. Since the PS and ML problems have been integrated in all the three classes of FMSs investigated, a set of consistent solutions for both PS and ML is the natural output of the models.

(5) Utilization of the Flexibility

As mentioned earlier, two of the most important benefits an FMS can offer are high productivity and flexibility. The models for the PS and ML problems should be so developed that the two benefits can be captured and, therefore, the restrictions imposed merely for convenience of computation or modelling but with negative effects on utilization of the two benefits should be removed as much as possible. For instance, the restrictions such as -- each machining center is designated to only one route, one type of tool can be assigned to only one machining center, one operation is allocated to only one machining center, etc. have certain negative effects on utilization of the flexibility of a FMS. In this research, these restrictions have been dropped. Some of the effects of these restrictions have been discussed.

(6) Some Insights on System Performance

As indicated by Hwang and Shogan (1989), the magazine capacity and available machining time are two important constraints which restrict system performance. For example, the number of parts which can be loaded to a system for simultaneously processing during the following period is restricted by the two constraints. In our study, it is observed that the number of parts that can be simultaneously loaded to

an FMS is also restricted by either one of the constraints. If one of the constraints, say, magazine capacity, is fixed, the number of parts which can be simultaneously loaded to the system initially increases with available machining time. But after a certain point (it may be called critical point), the number of parts that can be simultaneously loaded to the system will not increase with the available machining time. Any additional available machining time will become redundant. Similar observation can be made when we change magazine capacity for a fixed amount of available machining time.

It may therefore be inferred that for a given amount of available machining time (magazine capacity), there is a critical point below which the number of parts that can be simultaneously loaded to an FMS will increase with magazine capacity (available machining time). At this critical point the number of parts which can be simultaneously loaded to the system attains its maximum value. Beyond the critical point this value remains unchanged and additional magazine capacity (available machining time) does not create additional benefits.

8.2 Conclusions

In this dissertation, an integrated approach to part type selection and machining loading problems in flexible manufacturing systems was proposed to aid managers in making FMS planning related decisions and to avoid the possible inconsistency and conflicts caused by separately treating the two closely interrelated problems, as presented in most previous researches. Based on this approach, the combined part type selection and machine loading problems have been investigated in three

types of FMSs:

FMS I: A class of FMS where each machining center has a relatively small tool magazine, tool transportation devices are available, production batches are generally small.

FMS II: A type of FMS where each machining center has a relatively large tool magazine, tool transportation facilities are not available or not necessary, production batch sizes could be large and unstable.

Hybrid FMS: A manufacturing system in which conventional manufacturing system (CMS), such as job shop and mass production shop, and flexible manufacturing facilities, e.g., flexible manufacturing cells, or 'island of automations' operate simultaneously. The FMS sub-system could be either FMS I type or FMS II type. Two cases have been studied for this type of system: case 1, the CMS portion has sufficient capacity to process all production requirements that could not be finished by the FMS portion alone within the planning horizon; case 2, the production requirements of the parts ordered can not be finished by both FMS and CMS sub-systems within the planning horizon.

The following have been accomplished:

(1) Bicriterion models have been developed for the combined PS and ML problems in the three classes of FMSs to maximize system outputs (or to maximize savings by properly sharing production tasks between FMS and CMS sub-systems--in case 1 of hybrid FMS) and reduce system

inputs.

These models, once solved, will yield consistent solutions to PS and ML decisions, including part type selection, operation and/or proportion of operation allocation, tool type selection and allocation, process route selection and determination of number of spare tools of each type of tool on each machining center (in FMS II). Some less practical assumptions applied in most of previous research, such as

- . each operation can be assigned to only one machine;
 - . each type of tool can be allocated to only one machining center;
 - . each machining center is dedicated to one or few predetermined routes; and
 - . each type of operation is performed using only one type of tool;
- have been relaxed in these models to enhance utilization of system flexibility.

(2) The applications of the models have been illustrated using examples. The results show that the secondary objectives, such as processing cost or makespan, can be improved by rearranging job and tool allocation patterns without losing the optimality of the primary objective. This demonstrates that greater economic benefits can be realized in an FMS through enhanced utilization of flexibility inherent in the systems, provided the methodology suggested in the research is followed by the decision makers. The effects of tool magazine capacity and available machining time on primary objective (throughput, in the examples) were also investigated.

(3) Solution procedures for the models of FMS I, II and the FMS portion of the hybrid system have been suggested to solve relatively large sized problems. The solution procedures are based on Lagrangian relaxation and decomposition principle incorporating revised simplex method and column generation method.

(4) The implementation of the solution procedures is demonstrated by solving Model 1. Associated FORTRAN programs are created. Several sets of randomly generated problems have been tested. All of the test problems reach accepted percentage error within reasonable CPU time.

The implementation of the solution procedures of other models can be achieved by extending the FORTRAN program to incorporate the characteristic of each model.

(5) The model for the machine loading problem in the CMS sub-system of the hybrid system was developed separately, which can be applied to 'pure' conventional manufacturing systems. This model also relaxed the commonly used assumption that each operation can be assigned to only one machine. In this model, the most important production cost components: machining cost, material handling cost, setup cost and machine idle cost, are considered. In addition, by assigning penalty cost for long makespans, the optimal process routes for each part and optimal load for each machine in terms of minimum production cost can be obtained with lower makespan. An example problem is solved. The discussions with regard to the impact of batch splitting, setup cost

and demand change on optimal solution were also presented.

8.3 Suggestions for Future Research

This research was conducted for some specified FMSs based on certain assumptions. Further efforts may be worthwhile to extend the research based on the proposed integrated approach. The following extensions could be promising:

(1) Partial Lots Selection Consideration

In all of the models developed in this thesis, it is assumed that a type of part is either entirely selected or totally rejected for immediate processing. In practice, however, it may be desirable to select only proportions of lots of parts for immediate and simultaneous processing while leaving the remaining for future production to improve some desired system performance measures.

This type of problem can be modeled by dropping the integrality of variable X_p in the proposed models and adding an upper bound like constraint: $X_p \leq 1$. The solution method can also be adopted with some modifications.

(2) Material Handling Considerations in FMS

In all models developed for FMS I, II and the FMS sub-system in the hybrid system, an implicit assumption is that material handling capacity is sufficient or the associated cost is not important in comparison with the machining cost. In some cases, the material handling equipment may become a bottleneck and therefore, related

constraint should be added. Or, if material handling cost is not negligible in total production cost, it should be incorporated in a cost related secondary objective.

(3) Tooling Related Considerations

Two issues may be further investigated:

(a) In certain situations, the number of available spare tools of each type, or some type, of tools may be limited or restricted by tooling budget. In this case, an additional constraint has to be considered.

(b) In some FMSs, some types of N/C machines may require fine tuning during tool changes. The time needed for tuning operations is significantly related to the job processing time (Tang and Denardo, 1988). In this case, minimization of the number of tool changes may be considered as a secondary objective to reduce total throughput time.

REFERENCES

- Ammons, J.C., Lofgren, C.B., and McGinnis, L.F., 1985, A large scale machine loading problem in flexible assembly. *Annals of Operations Research*, Vol.3, 319-332.
- Arbel, A. and Seidmann, A., (1984), Performance evaluation of flexible manufacturing systems. *IEEE Transactions on Systems, Man and Cybernetics*, Vol.14, No.4, 606-612.
- Avont, L.H., Gelders, L.F., and Van Wassenhove, L.N.V., 1988, Allocating work between an FMS and a conventional jobshop: a case study. *European Journal of Operational Research*, Vol. 33, 245-256.
- Avont, L.H., Wassenhove, L.N.V., 1988, The part mix and routing mix problem in FMS: a coupling between an LP model and a closed queueing network. *International Journal of Production Research*, Vol.26, No.12, 1891-1902.
- Bazaraa, M.S., and Jarvis, J.J., 1977, *Linear Programming and Network Flow*, John Wiley & Sons, Inc., New York.
- Berrada, M., and Steckel, K.E., 1984, A branch and bound approach for FMS machine loading. *Proceedings of the first ORSA/TIMS Conference on FMS*, University of Michigan, Ann Arbor, Michigan, USA, 256-271.
- Bessant, J., and Hayward, B., 1986, FMS in Britain-good and bad news. *The FMS Magazine*, January, 35-39.
- Bretthauer, K.M., and Venkataramanan, M.A., 1990, Machine loading and alternate routing in a flexible manufacturing system. *Computers and Industrial Engineering*, Vol.18, No.3, 341-350.
- Browne, J., Dubois, D., Rathmill, K., Sethi, S.P., and Stacke, K.E., 1984, Classification of flexible manufacturing systems. *The FMS Magazine*, April, 114-117.
- Chakravarty, A.K., and Shtub, A., 1984, Selecting parts and loading flexible manufacturing systems. *Proceedings of the first ORSA/TIMS Conference on FMS*, University of Michigan, Ann Arbor, Michigan, USA, 284-289.
- Chen, Y., and Askin, R.G., 1990, A multiobjective evaluation of flexible manufacturing system loading heuristics. *International Journal of Production Research*, Vol.28, No.5, 895-911.
- Chung, C., 1986, Loading flexible manufacturing systems: a heuristic approach. *Computers and Industrial Engineering*, Vol.11, 246-250.
- Co, H.C., 1984, Design and implementation of FMS, Ph.D. Dissertation, Virginia Polytechnic Institute and State University, USA.
- Co, H.C., Bleimann, J.S., and Chen, S.K., 1990, A methodical approach to the flexible-manufacturing-system batching, loading and tool configuration problems. *International Journal of Production Research*,

Vol.28, No.12, 2171-2186.

ElMaraghy, H.A., 1985, Automated tool management in flexible manufacturing. *Journal of Manufacturing Systems*, Vol.4, No.1, 1-13.

Fisher, M.L., 1981, The Lagrangian relaxation method for solving integer programming problems. *Management Science*, Vol.27, No.1, 1-18.

Fisher, M.L., 1985, An applications oriented guide to Lagrangian relaxation. *Interfaces*, Vol.15, No.2, 10-21.

Gaimon, C., 1985, The dynamic optimal acquisition of automation. *Annals of Operations Research*, Vol.3, 59-79.

Gongware, T.A., and Ham, I., 1981, Cluster analysis applications for GT manufacturing systems. *Proceedings of IX th North American Metal Working Research Conference*, SME, Dearborn, Michigan, 503-508.

Gray, A.E., Seidmann, A., and Stecke, K.E., 1988, Tool management in automated manufacturing: operational issues and decision problems, working paper CMOM 88-03, University of Rochester.

Groover, M.P., 1980, *Automation, Production Systems, and Computer-Aided Manufacturing*, Prentice-Hall, Englewood Cliffs, NJ.

Gupta, D., Buzacott, J.A., and Gerchak, Y., 1988, Economic analysis of investment decisions in flexible manufacturing systems. Department of Management Science, University of Waterloo, Canada.

Han, C., and Ham, I., 1986, Multiobjective cluster analysis for part family formulations. *Journal of Manufacturing Systems*, Vol. 5, No.4, 223-230.

Han, M., Na, Y.K., and Hogg, G.L., 1989, Real-time control and job dispatching in flexible manufacturing systems. *International Journal of Production Research*, Vol.27, No.8, 1257-1267.

Hughes, J., 1987, It's methodology, not technology, that counts. *The FMS Magazine*, Vol.5, No.2, 81-85.

Hutchinson, G.K., and Holland, J.R., 1982, The economic value of flexible automation. *Journal of Manufacturing Systems*. Vol. 1, 215-227.

Hwang, S.S., 1986, A constraint-directed method to solve the part selection problem in flexible manufacturing systems planning stage. *Proceedings of the second ORSA/TIMS Conference on FMS*, Vol.5, University of Michigan, Ann Arbor, Michigan, USA, 297-309.

Hwang, S.S., and Shanthikumar, J.G., 1987, An FMS Production Planning System and Evaluation of Part selection Approaches, Management Science Working Paper No. MS-43, School of Business Administration, University of California, Berkeley.

Hwang, S.S., and Shogan, A.W., 1989, Modeling and solving an FMS part

selection problem. *International Journal of Production Research*, Vol.27, No.8, 1349-1366.

Ito, Yoshimi, 1981, Japanese FMS-present and a future review, Handout of Seminar on FMS, R.O.C.

Jain, A.K., Kasiliangam, R.G., and Bhole, S.D., 1991, Joint consideration of cell formulation and tool provisioning problems in flexible manufacturing systems. *Computers and Industrial Engineering*, Vol 20, No.2, 271-277.

Kayaligil, M.S., 1990, Part-mix allocation between automated and conventional manufacturing. *International Journal of Production Research*, Vol.28, No.10, 1847-1859.

Kochan, A., 1985, FMS is for the big boys-in the US. *The FMS Magazine*, July, 134-135.

Kumar, P., Singh, N., and Tewari, N.K., 1987, Multicriterion analysis of the loading problem in flexible manufacturing systems using min-max approach. *International Journal of Advanced Manufacturing Technology*, Vol.2, No.2, 13-23.

Kusiak, A., 1983a, Loading models in flexible manufacturing systems. *Proceedings of the VII th International Conference on Production Research*, Windsor, Ontario, Canada, 641-647.

Kusiak, A., 1983b, Loading models in FMS, recent developments in FMS and allied areas. in *The Design and Operation of FMS* (New York: North-Holland Publishing Co.).

Kusiak, A., 1984, The part families problem in flexible manufacturing systems. *Proceedings of the first ORSA/TIMS Conference on FMS*, University of Michigan, Ann Arbor, Michigan, USA, 237-242.

Kusiak, A., 1985a, Integer programming approach to process planning. *International Journal of Advanced Manufacturing Technology*, Vol.1, 73-83.

Kusiak, A., 1985b, Flexible manufacturing systems: a structural approach. *International Journal of Production Research*, Vol.23, No.6, 1057-1073.

Kusiak, A., 1985c, The part families problem in FMS. *Annals of Operations Research*, Vol.3, 279-300.

Kusiak, A., 1987, Artificial intelligence and operations research in flexible manufacturing systems. *Infor*, Vol.25, No.1, 2-12.

Lashkari, R. S., Dutta, S. P., and Padhye, A. M., 1987, A new formulation of operation allocation problem in flexible manufacturing systems: mathematical modelling and computational experience. *International Journal of Production Research*, Vol.25, No.9, 1267-1283.

Liang, M., and Dutta, S.P., 1990, A mixed-integer programming approach to the machine loading and process planning problem in a process layout environment. *International Journal of Production Research*, Vol.28, No.8, 1471-1484.

Martello, S., and Toth, P., 1978, Algorithm for the solution of the 0-1 single knapsack problem. *Computing*, Vol.21, No.1, 81-86.

Martello, S., and Toth, P., 1990, An exact algorithm for large unbounded knapsack problems. *Operations Research Letters*, 9, 15-20.

O'Grady, P.J., and Menon, U., 1987, Loading a flexible manufacturing system. *International Journal of Production Research*, Vol.25, No.7, 1053-1068.

Rajagopalan, S., 1986, Formulation and heuristic solutions for parts grouping and tool loading in flexible manufacturing systems. *Proceedings of the second ORSA/TIMS Conference on FMS*, Vol.5, University of Michigan, Ann Arbor, Michigan, USA, 477-486.

Ram, B., Sarin, S., and Chen, C.S., 1990, A model and a solution approach for the machine loading and tool allocation problem in a flexible manufacturing system. *International Journal of Production Research*, Vol.28, No.4, 637-645.

Ranky, P., 1983, *The Design and Operation FMS*, IFS (Publication) Ltd., UK.

Sarin, S.C., and Chen, C.S., 1987, The machine loading and tool allocation problem in a flexible manufacturing system. *International Journal of Production Research*, Vol.25, No.7, 1081-1094.

Shanker, K.S., and Srinivasulu, A., 1989, Some solution methodologies for loading problems in a flexible manufacturing system. *International Journal of Production Research*, Vol.27, No.6, 1019-1034.

Shanker, K.S., and Tzen, Y.J., 1985, A loading and dispatching problem in a random flexible manufacturing system. *International Journal of Production Research*, Vol.23, No.3, 579-595.

Smith, M.L., Ramesh, R., Dudek, R.A., and Blair, E.L., 1986, Characteristics of U.S. flexible manufacturing systems -- A survey. *Proceedings of the second ORSA/TIMS Conference on FMS*, Vol.5, University of Michigan, Ann Arbor, Michigan, USA, 477-486.

Stecke, K.E., 1983, Formulation and solution of nonlinear integer production planning problems for flexible manufacturing systems. *Management Science*, Vol.29, No.3, 273-288.

Stecke, K.E., 1984, Design, planning, scheduling, and control problems of flexible manufacturing systems. *Proceedings of the First ORSA/TIMS Conference on FMS*, University of Michigan, Ann Arbor, Michigan, USA, 1-7.

Stecke, K.E., and Kim, I., 1986, A flexible approach to implementing

the short-term FMS planning function. *Proceedings of the Second ORSA/TIMS Conference on FMS*, University of Michigan, Ann Arbor, Michigan, USA, 283-295.

Stecke, K.E., and Kim, I., 1987, Comparison of various approaches to part type selection problem in flexible manufacturing systems. *Proceedings of the IXth International Conference on Production Research*, 2275-2281.

Stecke, K.E., and Solberg, J.J., 1981, Loading and control policies for a flexible manufacturing system. *International Journal of Production Research*, Vol.19, No.5, 481-490.

Szadkowski, J., 1971, An approach to machining process optimization. *International Journal of Production Research*, Vol.9, No.3, 371-376.

Tang, S., and Denardo, E.V., 1988, Models arising from a flexible manufacturing machine, part I: minimization of the number of tool switches. *Operations Research*, Vol.36, No.5, 767-778.

Tomek, P., 1986, Tooling strategies related to FMS management. *The FMS Magazine*, April, 102-107.

Whitney, C.K., and Gaul, T.S., 1984, Sequential decision procedures for batching and balancing in FMSs. *Proceedings of the first ORSA/TIMS Conference on FMS*, University of Michigan, Ann Arbor, Michigan, USA, 243-248.

Whitney, C.K., and Suri, R., 1984, Decision aids for FMS part and machine selection. *Proceedings of the First ORSA/TIMS Conference on FMS*, University of Michigan, Ann Arbor, Michigan, USA, 205-210.

APPENDIX 1.

DECOMPOSITION AND COLUMN GENERATION PROCEDURE FOR SUB-PROBLEM SUB1-1 IN CHAPTER 3

The relaxed Sub1-1 can be written in the following vector form:

$$\text{Min. } C_1 G_1 + C_2 G_2 + \dots + C_p G_p$$

subject to

$$A_1 G_1 + A_2 G_2 + \dots + A_p G_p \leq a \quad (3-3')$$

$$\begin{array}{rcl} B_1 G_1 & \leq & b_1 \\ B_2 G_2 & \leq & b_2 \\ & \cdot & \\ & \cdot & \\ B_p G_p & \leq & b_p \end{array} \quad (3-2')$$

where

$$C_p = (u_{11}, u_{12}, \dots, u_{1K}, u_{21}, u_{22}, \dots, u_{2K}, \dots, u_{T1}, \dots, u_{TK}, \leftarrow j = 1$$

$$u_{11}, u_{12}, \dots, u_{1K}, u_{21}, u_{22}, \dots, u_{2K}, \dots, u_{T1}, \dots, u_{TK}, \leftarrow j = 2$$

.....

$$u_{11}, u_{12}, \dots, u_{1K}, u_{21}, u_{22}, \dots, u_{2K}, \dots, u_{T1}, \dots, u_{TK}, \leftarrow j = J_p$$

$$-W_p)$$

$$G_p = (X_{p111}, \dots, X_{p11K}, X_{p121}, \dots, X_{p12K}, \dots, X_{p1T1}, \dots, X_{p1TK}, \leftarrow j = 1$$

$$X_{p211}, \dots, X_{p21K}, X_{p221}, \dots, X_{p22K}, \dots, X_{p2T1}, \dots, X_{p2TK}, \leftarrow j = 2$$

.....

$$X_{pJ_{11}}, \dots, X_{pJ_{1K}}, X_{pJ_{21}}, \dots, X_{pJ_{2K}}, \dots, X_{pJ_{T1}}, \dots, X_{pJ_{TK}}, \leftarrow j = J_p$$

$$X_p)^T \text{ and satisfies constraints (3-17) and (3-8).}$$

$$A_p = \begin{pmatrix} \overbrace{t_{p111} \quad 0 \quad t_{p121} \quad 0 \quad \dots \quad t_{p1T1}}^{j=1} \\ t_{p112} \quad t_{p122} \quad \dots \quad t_{p1T2} \\ \vdots \quad \vdots \quad \dots \quad \vdots \\ 0 \quad t_{p11K} \quad 0 \quad t_{p12K} \quad \dots \quad t_{p1TK} \end{pmatrix}$$

.....

$$\begin{array}{c}
 j = J_p \\
 \left(\begin{array}{cccccc}
 t_{pJ,11} & 0 & t_{pJ,21} & 0 & t_{pJ,T1} & 0 \\
 & t_{pJ,12} & & t_{pJ,22} & & t_{pJ,T2} \\
 & & \ddots & & \ddots & \\
 0 & & & t_{pJ,1K} & 0 & t_{pJ,2K} \\
 & & & & & t_{pJ,TK}
 \end{array} \right)
 \end{array}$$

$$B_p = \left(\begin{array}{cccccc}
 \overbrace{1,1,\dots,1}^{j=1} & \overbrace{0,0,\dots,0}^{j=2} & \dots & \overbrace{0}^{j=J_p} & -1 \\
 & \overbrace{1,1,\dots,1}^{j=2} & & & -1 \\
 & & \ddots & & \vdots \\
 0 & & & & \vdots \\
 & & & \overbrace{1,1,\dots,1}^{j=J_p} & -1 \\
 0, \dots, \dots, 0, \dots, 0, & & & & 1
 \end{array} \right) \leftarrow J_p + 1 \text{ th row}$$

$$a = \begin{pmatrix} A_1 \\ A_2 \\ \vdots \\ A_k \end{pmatrix} \qquad b = \begin{pmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 1 \end{pmatrix} \leftarrow \text{position } J_p + 1$$

This is an block diagonal structure. Each block represents a particular part p and has a generalized upper bound (GUB) like structure (Fisher, 1981) which can be easily solved using inspecting methods. According to the decomposition principle, the problem can be decomposed into two separate problems: one over constraint (3-3') is called the master problem and one over constraint (3-2') is called the subproblem.

Before presenting the solution procedure, sub1-1 is reformulated. Let $\Omega_p = \{ G_p : B_p G_p \leq b_p, G_p \text{ satisfies constraints (3-17), (3-8) } \}$ for $p=1,2,\dots,P$ and replacing G_p by

$$\sum_{m=1}^{M_p} \lambda_{pm} G_{pm}$$

where
$$\sum_{m=1}^{M_p} \lambda_{pm} = 1$$

$$\lambda_{pm} \geq 0 \quad m = 1, 2, \dots, M_p$$

G_{pm} -- extreme point of Ω_p

M_p -- number of extreme points of the set Ω_p

Then the foregoing optimization problem sub1-1 can be transformed as follows:

Model (MD-1)

$$\text{Min. } \sum_{p=1}^P \sum_{m=1}^{M_p} (C_p G_{pm}) \lambda_{pm} \quad (3-18)$$

subject to

$$\sum_{p=1}^P \sum_{m=1}^{M_p} (A_p G_{pm}) \lambda_{pm} + S = a \quad (3-19)$$

$$\sum_{m=1}^M \lambda_{pm} = 1 \quad \forall p \quad (3-20)$$

$$\lambda_{pm} \geq 0 \quad \forall p, m$$

where $S = (s_1, s_2, \dots, s_k, \dots, s_k)^T$, a slack variable vector.

This problem is solved using the revised simplex method and column generation scheme (Bazarrá, 1977), which involves the following main steps.

Algorithm MD-1

step 1. (Initialize)

Set $m = 1$. Find an initial basic feasible solution of model (M-D) by letting $G_{p1} = 0$ ($p = 1, 2, \dots, P$), which is a convenient feasible solution since each $G_{p1} = 0$ is an extreme point of Ω_p . Then we have a basis consists of S and λ_{p1} ($p=1, 2, \dots, P$) and a basis inverse $B^{-1} = I$ (a $(K+P) \times (K+P)$ identity matrix). Denoting the dual variables corresponding to (3-19) and (3-20) by $\pi = (\pi_1, \pi_2, \dots, \pi_k)$ and $\alpha =$

$(\alpha_1, \alpha_2, \dots, \alpha_P)$, we get $(\pi, \alpha) = \hat{C}_B^T B^{-1}$, where \hat{C}_B is a vector with $\hat{C}_{pm} = C_{pm}$ for each basic variable λ_{pm} . Therefore, the initial tableau is formed as follows.

RHS		
Z	$(\pi, \alpha) = (0, 0)$	$\hat{C}_B \bar{a} = 0$
S	B^{-1}	a
λ		1

where $\bar{a} = B^{-1} \begin{pmatrix} a \\ 1 \end{pmatrix}$, $1 = (1, 1, \dots, 1)^T$, a $1 \times P$ vector.

step 2. (check improvability)

At iteration m, check if the optimal solution is obtained or further improvement is possible. This is done by solving the following P subproblems.

Vector form:

$$\text{Max. OBJ}_p = (\pi A_p - C_p) G_p + \alpha_p \quad p=1, \dots, P$$

subject to

$$G_p \in \Omega_p \quad p=1, \dots, P$$

Scalar form:

$$\text{Max. OBJ}_p = \sum_j \sum_t \sum_k (\pi_k t_{pjtk} - u_{tk}) X_{pjtk} + W_p X_p + \alpha_p$$

subject to

$$\sum_p \sum_j X_{pjtk} - X_p = 0 \quad \forall j, p=1, \dots, P$$

$$0 \leq X_p \leq 1$$

As mentioned above, each of the P subproblems has general upper bound (GUB) like constraints. Such a problem is easily solved by determining $\delta_{pj} = \max_{tk} (\pi_k t_{pjtk} - u_{tk})$ for each j and setting the corresponding $X_{pjtk} = 1$ and $X_p = 1$, if $\sum_j \delta_{pj} + W_p \geq 0$ and remaining $X_{pjtk} = 0$; if $\sum_j \delta_{pj} + W_p < 0$, let $X_{pjtk} = 0$ ($\forall j, t, k$) and $X_p = 0$.

Let G_{pm} be the optimal basic feasible solution of subproblem p with objective value OBJ_{pm} . If all the objectives of the P subproblems are 0 then the optimal solution is reached. Otherwise, check all π_k , if $\pi_k > 0$, the associated slack variable is eligible to enter basis. If there are more than one π_k great than 0, we select the slack variable corresponding to most positive π_k to enter master basis. (This is because that all constraints in following mentioned master problem are \leq type). If there is no π_k great than 0 we select the λ_{pm} associated with the highest OBJ_{pm} to enter master basis.

If $OBJ_p (\forall p) = 0$, then stop, the optimal solution is obtained; else, select the λ_{pm} associated with most positive OBJ_p to enter basis and go to step 3.

step 3 (Generate new column)

Let $YC_{pm} = B^{-1} \begin{pmatrix} A_p & G_{pm} \\ 0 \\ 0 \\ \cdot \\ 0 \\ 1 \\ 0 \\ \cdot \\ 0 \end{pmatrix}$ \leftarrow position $K + p$

Then the updated column is formed as $\begin{pmatrix} OBJ_{pm} \\ YC_{pm} \end{pmatrix}$

step 4 (update master basis)

Pivot at YC_{pm}^r where the index r is determined by:

$$\frac{\bar{a}_r}{YC_{pm}^r} = \text{Minimize}_{1 \leq n \leq K+p} \left\{ \frac{\bar{a}_n}{YC_{pm}^n} : YC_{pm}^n > 0 \right\}$$

This updates the basis inverse, the dual variables, and the right-hand side. Then go to step 2.

APPENDIX 2

DECOMPOSITION AND COLUMN GENERATION PROCEDURE

FOR SUB-PROBLEM SUB 1-3 IN CHAPTER 3

For convenience of presentation in Appendix 2--3, denote I as the total number of selected part types.

The vector form of sub1-3 is

$$\text{Min. } C_1 G_1 + C_2 G_2 + \dots + C_I G_I$$

subject to

$$A_1 G_1 + A_2 G_2 + \dots + A_I G_I \leq a \quad (3-11')$$

$$\begin{aligned} B_1 G_1 &= b_1 \\ B_2 G_2 &= b_2 \\ &\vdots \\ B_I G_I &= b_I \end{aligned} \quad (3-10')$$

where

$$C_j = ((C_{111} + u_1)^T, (C_{112} + u_2)^T, \dots, (C_{11T} + u_T)^T, \quad \leftarrow j = 1$$

$$(C_{121} + u_1)^T, (C_{122} + u_2)^T, \dots, (C_{12T} + u_T)^T, \quad \leftarrow j = 2$$

.....

$$(C_{1J_1 1} + u_1)^T, (C_{1J_1 2} + u_2)^T, \dots, (C_{1J_1 T} + u_T)^T \quad \leftarrow J = J_1$$

where

$$C_{1jt} = (C_{1jt1}, C_{1jt2}, \dots, C_{1jtK})^T \quad \forall 1, j, t$$

$$u_t = (u_{t1}, u_{t2}, \dots, u_{tK})^T \quad \forall t$$

$$G_1 = \begin{pmatrix} \bar{X}_{1111}, \dots, \bar{X}_{111K}, \bar{X}_{1121}, \dots, \bar{X}_{112K}, \dots, \bar{X}_{11T1}, \dots, \bar{X}_{11TK}, \\ \bar{X}_{1211}, \dots, \bar{X}_{121K}, \bar{X}_{1221}, \dots, \bar{X}_{122K}, \dots, \bar{X}_{12T1}, \dots, \bar{X}_{12TK}, \\ \dots\dots\dots \\ \bar{X}_{1J_1 11}, \dots, \bar{X}_{1J_1 1K}, \bar{X}_{1J_1 21}, \dots, \bar{X}_{1J_1 2K}, \dots, \bar{X}_{1J_1 T1}, \dots, \bar{X}_{1J_1 TK} \end{pmatrix}$$

and satisfies constraints (3-13).

$$A_1 = \begin{pmatrix} \overbrace{\bar{t}_{1111} \quad 0 \quad \bar{t}_{1121} \quad 0 \quad \bar{t}_{11T1}}^{j=1} \\ \bar{t}_{1112} \quad \quad \bar{t}_{1122} \quad \quad \bar{t}_{11T2} \\ \vdots \quad \quad \quad \vdots \quad \quad \quad \vdots \\ 0 \quad \bar{t}_{111K} \quad 0 \quad \bar{t}_{112K} \quad \bar{t}_{11TK} \end{pmatrix}$$

$$\begin{array}{c}
 \dots\dots\dots \\
 j = J_1 \\
 \left(\begin{array}{cccc}
 \overbrace{\bar{t}_{1J_1 11} \quad 0 \quad \bar{t}_{1J_1 21} \quad 0 \quad \bar{t}_{1J_1 T1}} & 0 \\
 \bar{t}_{1J_1 12} & \bar{t}_{1J_1 22} & \bar{t}_{1J_1 T2} \\
 & \ddots & \ddots & \ddots \\
 0 & \bar{t}_{1J_1 1K} & 0 & \bar{t}_{1J_1 2K} & 0 & \bar{t}_{1J_1 TK}
 \end{array} \right)_1
 \end{array}$$

$$B_1 = \left(\begin{array}{ccc}
 \overbrace{1, 1, \dots, 1}^{j=1} & \overbrace{0, 0, \dots, 0}^{j=2} & \dots & \overbrace{0}^{j=J_1} \\
 & \overbrace{1, 1, \dots, 1}^{j=2} & & \\
 & 0 & \ddots & \\
 & & & \overbrace{1, 1, \dots, 1}^{j=J_1}
 \end{array} \right)$$

$$a = \begin{pmatrix} A_1 \\ A_2 \\ \vdots \\ A_k \end{pmatrix} \qquad b_1 = \left\{ \begin{pmatrix} 1 \\ 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix} \right\}_{J_1 \text{ "1"s}}$$

By using the decomposition principle, the problem can be decomposed into two separate problems: one over constraint (3-11') is called the master problem and one over constraint (3-10') is called the subproblem.

Sub1-3 is reformulated as follows.

Let $\Omega_i = \{ G_i : B_i G_i = b_i, G_i \text{ satisfies constraints (3-13)} \}$ for $i=1,2,\dots,I$ and replace G_i by

$$\sum_{m=1}^{M_i} \lambda_{im} G_{im}$$

where
$$\sum_{m=1}^{M_i} \lambda_{im} = 1$$

$$\lambda_{im} \geq 0 \quad m = 1, 2, \dots, M_i$$

G_{im} -- extreme point of Ω_i

M_i -- number of extreme points of the set Ω_i

Then the foregoing problem sub1-3 can be transformed as follows:

Model (MD1-1)

$$\text{Min. } \sum_{i=1}^I \sum_{m=1}^{M_i} (C_i G_{im}) \lambda_{im} \quad (3-18')$$

subject to

$$\sum_{i \in I_1} \sum_{m=1}^{M_i} (A_i G_{im}) \lambda_{im} + S = a \quad (3-19')$$

$$\sum_{m=1}^{M_1} \lambda_{1m} = 1 \quad i \in I_1 \quad (3-20')$$

$$\lambda_{1m} \geq 0 \quad i \in I_1, \forall m$$

where $S = (s_1, s_2, \dots, s_k, \dots, s_k)^T$, a slack variable vector.

This problem can be solved using similar procedures presented in Appendix 1. The details are given below.

Algorithm MD1-1

step 1. Initialize

Follow step 1 of algorithm MD-1. But here the dual vectors π and α are corresponding to (3-19') and (3-20'), respectively.

step 2. Check improvability

At iteration m , check if the optimal solution is obtained or if further improvement is possible by solving the following P subproblems.

Vector form:

$$\text{Max. OBJ}_i = (\pi A_i - C_i) G_i + \alpha_i \quad i \in I_1$$

subject to

$$G_i \in \Omega_i$$

Scalar form:

$$\text{Max. OBJ}_1 = \sum_j \sum_t \sum_k (\pi_k \bar{t}_{1jtk} - u_{tk} - C_{1jtk}) \bar{X}_{1jtk} + \alpha_1$$

subject to:

$$\sum_t \sum_k \bar{X}_{1jtk} = 1 \quad \forall j, 1 \in I_1$$

and (3-13).

Each of the P subproblems has general upper bound (GUB) like constraints and can be easily solved as follows.

Sub-step 1. Input u_{tk}^m , α_1^m , and π_k^m .

Sub-step 2. For a given set of u_{tk}^m , α_1^m , and π_k^m , let

$$\delta_{1j} = \max_{tk} (\pi_k \bar{t}_{1jtk} - u_{tk} - C_{1jtk}) \quad \forall j$$

If $\sum_j \delta_{1j} \geq 0$ then let associated $\bar{X}_{1jtk} = 1$, other $\bar{X}_{1jtk} = 0$. Then the optimal solution of each of the I subproblem is at hand with objective value

$$\text{OBJ}_{1m} = \sum_j \delta_{1j} + \alpha_1^m$$

If $\sum_j \delta_{1j} < 0$, let all $\bar{X}_{1jtk} = 0$.

Let G_{im} be the optimal basic feasible solution of subproblem i with objective value OBJ_{im} . If all the objectives of the I_1 subproblems are 0 then the optimal solution is reached. Otherwise, check all π_k , if $\pi_k > 0$, the associated slack variable is eligible to enter basis. If there are more than one π_k 's great than 0, we select the slack variable corresponding to most positive π_k to enter master basis. (This is because that all constraints in following mentioned master problem are \leq type). If there is no π_k great than 0 we select the λ_{im} associated with the highest OBJ_{im} to enter master basis.

step 3 - 4 Follow step 3 and 4 of algorithm MD-1.

APPENDIX 3

DECOMPOSITION AND COLUMN GENERATION PROCEDURE

FOR SUB-PROBLEM SUB 1-5 IN CHAPTER 3

In order to get none zero right hand side values for constraint (3-15), we add constant r_k to both sides of each of constraints (3-15). Further, by replacing H with $H' (=H-1)$, we have

$$\text{Min. } \sum_{i \in I_1} \sum_{j=1}^J \sum_{t=1}^T \sum_{k=1}^K u_{tk} \bar{X}_{ijtk} + H' - 1$$

Subject to

$$\sum_{i \in I_1} \sum_{j=1}^J \sum_{t=1}^T \bar{t}_{ijtk} \bar{X}_{ijtk} - H' r_k \leq r_k \quad \forall k \quad (3-15a)$$

$$H' \leq A-1 \quad (3-16a)$$

and (3-10), (3-13).

This problem can be solved by first solving

$$\text{Min. } \sum_{i \in I_1} \sum_{j=1}^J \sum_{t=1}^T \sum_{k=1}^K u_{tk} \bar{X}_{ijtk} + H'$$

subject to (3-15a), (3-16a), (3-10) and (3-13) and then adding 1 to the optimal objective value.

Similar to sub1-1 and sub1-3, sub1-5 can be written in the following vector form:

$$\text{Min. } C_1 G_1 + C_2 G_2 + \dots + C_I G_I + C_{I+1} G_{I+1}$$

subject to

$$A_1 G_1 + A_2 G_2 + \dots + A_I G_I + C_{I+1} G_{I+1} \leq a \quad (3-15')$$

$$\begin{array}{rcl} B_1 G_1 & & = b_1 \\ B_2 G_2 & & = b_2 \\ & \ddots & \\ B_I G_I & & = b_I \end{array} \quad (3-10')$$

$$B_{I+1} G_{I+1} \leq b_{I+1}$$

where

$$C_1 = (u_{11}, u_{12}, \dots, u_{1K}, u_{21}, u_{22}, \dots, u_{2K}, \dots, u_{T1}, \dots, u_{TK}, \leftarrow j = 1$$

$$u_{11}, u_{12}, \dots, u_{1K}, u_{21}, u_{22}, \dots, u_{2K}, \dots, u_{T1}, \dots, u_{TK}, \leftarrow j = 2$$

.....

$$u_{11}, u_{12}, \dots, u_{1K}, u_{21}, u_{22}, \dots, u_{2K}, \dots, u_{T1}, \dots, u_{TK}) \leftarrow j = J_1$$

$$C_{I+1} = (1)$$

$$j = J_1$$

$$\left(\begin{array}{cccc} \overline{t}_{1J_1 11} & 0 & \overline{t}_{1J_1 21} & 0 & \overline{t}_{1J_1 T1} & 0 \\ & t_{1J_1 12} & & t_{1J_1 22} & & t_{1J_1 T2} \\ & & \ddots & & \dots & \\ 0 & & & t_{1J_1 1K} & 0 & & t_{1J_1 2K} & 0 & & t_{1J_1 TK} \end{array} \right)$$

$$A_{I+1} = (-r_1, -r_2, \dots, -r_K)^T$$

$$B_1 = \left(\begin{array}{ccc} \overbrace{1, 1, \dots, 1}^{j=1} & \overbrace{0, 0, \dots, 0}^{j=2} & \dots & \overbrace{0}^{j=J_1} \\ & \overbrace{1, 1, \dots, 1}^{j=2} & & \\ & & \ddots & \\ 0 & & & \overbrace{1, 1, \dots, 1}^{j=J_1} \end{array} \right)$$

$$B_{I+1} = (1)$$

$$a = \left(\begin{array}{c} r_1 \\ r_2 \\ \vdots \\ r_k \end{array} \right) \quad b_1 = \left\{ \left(\begin{array}{c} 1 \\ 1 \\ 1 \\ \vdots \\ 1 \end{array} \right) \right\} \quad J_1 \text{ "1"s}$$

$$b_{I+1} = (A-1)$$

This is again an block diagonal structure. Each block represents a particular part i for $i=1, \dots, I$ and the last block corresponds to variable H' . Using the decomposition principle, the problem can be decomposed into two separate problems: one over constraint (3-15') is

called the master problem and one over constraint (3-10') is called the subproblem.

Sub1-5 is reformulated as follows:

Let $\Omega_i = \{ G_i : B_i G_i \leq b_i, G_i \text{ satisfies constraints (3-13)} \}$ for $i=1,2,\dots,I+1$ and replace G_i by

$$\sum_{m=1}^{M_i} \lambda_{im} = 1$$

where $\sum_{m=1}^{M_i} \lambda_{im} = 1$

$$\lambda_{im} \geq 0 \quad m = 1, 2, \dots, M_{I+1}$$

G_{im} -- extreme point of Ω_i

M_i -- number of extreme points of the set Ω_i

Then the foregoing optimization problem sub1-5 can be transformed as follows:

Model (MD-2)

$$\text{Min. } \sum_{i \in I_1} \sum_{m=1}^{M_i} (C_i G_{im}) \lambda_{im} \quad (3-18'')$$

subject to

$$\sum_{i \in I_1} \sum_{m=1}^{M_1} (A_{i,im} G_{i,im}) \lambda_{i,im} + S = a \quad (3-19'')$$

$$\sum_{m=1}^{M_1} \lambda_{i,im} = 1 \quad i=1, \dots, I+1 \quad (3-20'')$$

$$\lambda_{i,im} \geq 0 \quad \forall m, i=1, \dots, I+1$$

where $S = (s_1, s_2, \dots, s_k, \dots, s_k)^T$, a slack variable vector.

The algorithm to solve this problem is similar to the ones for Models MD-1 and MD1-1. The algorithm is as follows.

Algorithm MD-2

step 1 -- 2.

Follow step 1--2 of algorithm MD. But here the dual vector of constraint (3-20'') consists of $I+1$ components, namely,

$$\alpha = (\alpha_1, \alpha_2, \dots, \alpha_{I+1}) \text{ and there are } I+1 \text{ subproblems}$$

For $i=1, \dots, I$, we have

$$\text{Max. OBJ}_i = \sum_j \sum_t \sum_k (\pi_k t_{ijtk} - u_{tk}) \bar{X}_{ijtk} + \alpha_i$$

subject to (3-13) and

$$\sum_t \sum_k \bar{X}_{ijtk} = 1 \quad \forall j$$

For $i=i+1$, the subproblem is

$$\text{Max. } (-\sum_k \pi_k r_k - 1) H' + \alpha_{i+1}$$

subject to

$$H' \leq (A-1)$$

Each of the subproblems can be solved using the similar method as described in algorithm MD1-1:

Sub-step 1. Input u_{tk} , α_i , and π_k .

Sub-step 2. For a given set of u_{tk} , α_i , and π_k , if $i \in I_1$, let

$$\delta_{ij} = \max_{tk} (\pi_k \bar{t}_{ijtk} - u_{tk}) \quad \forall j$$

If $\sum_j \delta_{ij} \geq 0$ then let the associated $\bar{X}_{ijtk} = 1$, other $\bar{X}_{ijtk} = 0$. Then the optimal solution of each of the I subproblem is at hand with objective value

$$\text{OBJ}_{im} = \sum_j \delta_{ij} + \alpha_i$$

If $\sum_j \delta_{ij} < 0$, let all $\bar{X}_{ijtk} = 0$.

For $i=i+1$, let $H=A$, if $(-\sum_k \pi_k \Gamma_k - 1) > 0$; $H=0$, if $(-\sum_k \pi_k \Gamma_k - 1) \leq 0$.

The remaining of this step is the same as mentioned in step 2 of algorithm MD-1 and MD1-1.

Step 3--4 Same as Step 3-4 of MD-2.

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